S282 Astronomy

Filaments and prominences

Study time: 45 minutes

Summary

In this activity you will be using images of the Sun that have been obtained from two major solar observatories in California and Austria to investigate the features of the Sun's chromosphere. This image-based activity uses the Hα image sequence set from 'The Sun' section of the Image Archive.

You will also need: a piece of thin string at least 30 cm long and a 30 cm ruler.

You should have read to the end of Section 1.3 of *An Introduction to the Sun and Stars* before starting this activity.

Learning outcomes

- Understand that prominences and filaments are features of the chromosphere.
- Understand that prominences and filaments are studied from images taken at the wavelength of certain spectral lines, and in particular, the Hα line.
- Appreciate the physical length scales associated with filaments.
- Appreciate the transient nature of filaments.

The activity

- Start the S282 Multimedia guide and open the folder called 'The Sun', then click on the icon for this activity ('Filaments and prominences').
- Press the Start button to launch the Image Archive at the required set of images.
- Alternatively, launch the Image Archive by clicking the Image Archive button in the Multimedia guide and then find the 'H-alpha images' sequence set which is located within 'The Sun' section of the archive.
- Place the cursor over one of the thumbnail images at the bottom of the screen – the date of the observation should appear.

A quick look at any of the images in the Hα image set will show you that this is not a familiar view of the Sun. Both observatories use digital cameras known as charge-coupled devices (CCDs) to obtain these images, so the only disadvantage in viewing them in the comfort of your own home rather than travelling to the observatory to view them there is that we have reduced them in size to make them more manageable (unless of course you wanted a trip to Austria or California!)

■ What is the range of dates that these observations cover (you will need to scroll through the thumbnails using the arrows below and/or to the side of them).

❑ The daily images range over the two-month period 1 April 2002 to 3 June 2002.

■ Why do you think there are no images between 9–21 April?

❑ The atmospheric conditions weren't good enough to produce sufficiently high quality images.

It may have been cloudy (a condition only too familiar to astronomers from damp temperate countries) or it may be that there was too much turbulence in the atmosphere to produce an image that doesn't appear to be smeared or out of focus. The observatory sites have been carefully chosen to minimize the number of days when images can't be taken.

The Big Bear Solar Observatory (BBSO) was built in 1969 in the middle of Big Bear Lake, California, at a height of 2000 m. This is at a higher altitude than most of Europe north of the Alps.

■ Why do you think the observatory is located at high altitude?

❑ Observatories tend to be sited high up because the atmosphere at such sites is clearer and thinner.

Compared to other astronomical observatories, solar observatories have an extra problem because they are in use during the day. You have probably seen heat haze rising off a hot road on a clear sunny day and the shimmering effect it produces as the air moves. This is exactly what we need to avoid if we are to obtain clear images of the Sun. At the BBSO the lake's water heats up much more slowly than the ground and so the images are less distorted by this effect. Turbulent motions in the air near the observatory are also reduced by the smooth flow of the wind across the lake instead of the turbulent flow that occurs over mountain peaks and forests.

The observatory has four telescopes which are specially designed for solar observations: a 65 cm reflector and three smaller refractors of 25, 20 and 15 cm. The whole Sun images you will be using are taken with the 20 cm telescope, which observes the Sun from sunrise to sunset each clear day, obtaining an image every 30 seconds.

The Kanzelhöhe Solar Observatory was founded during World War II by the German Luftwaffe near Villach in the Austrian Alps, to research the effects of the Sun on the part of the atmosphere known as the ionosphere. It is now part of the University of Graz. Like BBSO they have an array of small telescopes continuously monitoring the Sun, as well as a larger one doing more specialized work. The whole Sun Hα images are produced at one minute intervals during the hours of daylight.

(If you want to find out more about these two facilities, you can visit the websites of the BBSO (www.bbso.njit.edu) and the Kanzelhöhe Observatory (www.solobskh.ac.at/index_en.php). Note, however, that visits to these websites are not included in the study time for this activity.)

Question 1

What do we mean by an Hα image, and why do we use them to investigate the chromosphere?

- Scan along the thumbnail images until you reach the image for 20/05/02.
- Click on the thumbnail to open the image.
- First take a good look at the whole picture, but note that to investigate the details properly you should expand it to full-size by clicking on the button above it.

Question 2

Look carefully at this image, noting any important large-scale features. Bearing in mind what you read in Section 1.2 of *An Introduction to the Sun and Stars*, do you notice anything surprising?

Question 3

Make a rough sketch of the image, labelling those features you can identify. Describe in detail any features that you can't identify.

You estimated the size of a plage in Question 1.13 in *An Introduction to the Sun and Stars*. You are now going to estimate the typical length of filaments by measuring them on the image.

Prepare your scale by using your ruler to measure the radius of the Sun on your image.

- Use the full-size image as it will be easier to measure the filaments.
- A useful tip for making available as much screen space as possible for viewing the high resolution image is as follows. Right-click on the link that says 'Click here for full-size image'. In the menu that appears, there should be an option to 'Open in New Window'. Click on this, and you will see the large-scale image open in its own window – a window that can be as large as the available screen area.
- If you can measure the whole diameter directly from screen, so much the better. Remember to move your head so that your line of sight is at right angles to the screen at each end of the diameter. Divide the diameter by two to find the extent of the solar radius on your screen.
- If you can't get the whole width of the image of the Sun on the screen, you will have to estimate where the centre of the image is and measure from there to the edge. Don't forget to estimate the uncertainty in this measurement.

For example, on my screen I displayed the full-size image as described using the tip given above. My screen isn't large enough to see the entire solar disc, but it does show the whole width of the Sun. By scrolling the image down a little, I could easily see the widest part of the solar image. This distance, of course, corresponds to the solar diameter, which I measured on my screen to be (250 ± 4) mm. Thus the extent of the solar radius on my computer screen is (125 ± 2) mm.

- Now label the long filament across the left-hand side of the southern hemisphere on your sketch as filament ‘A’ (as in Figure 2). Then choose five other filaments and mark them B–F on your sketch. (These don’t have to be the same filaments as marked in Figure 2.) You will have to use your judgement to decide where one filament stops and another begins. Try not to choose the six longest (or shortest), but select some long and some short ones.
- Measure the lengths of these filaments using the string to trace out their extent on the screen (not your sketch) and then measuring the whole length of string against the ruler. Remember to estimate the uncertainty in your measurements.

Question 4

(a) Use the solar radius scale you measured earlier to calculate the lengths of the six filaments A–F *as they appear on the solar disc*. Tabulate your results. (Note that the solar radius is $R_{\odot} = 6.96 \times 10^5$ km.)

(b) How long are the shortest and longest measured filaments?

(c) There are two reasons why the lengths that you have measured may be *underestimates* of the true lengths of these features. What are these two reasons?

As highlighted in the answer to Question 4, you have not yet taken account of any foreshortening of features that occurs when a filament lies close to the solar limb – you have measured the lengths of filaments as they appear in projection. Near the centre of the solar disc, we see the filament from directly above and its apparent length will be a good estimate of its actual length. If we measure filaments away from the centre of the solar disc the apparent length will be substantially shorter than the true length. It is possible to make corrections for this projection effect, and convert any apparent lengths into actual lengths. It would be a lengthy diversion to do this fully, since the conversion factor depends not only on the location of the filament on the solar disc but also on its orientation. For our purposes we can use an approximate correction to the apparent length of features at different locations on the solar disc as shown in Figure 1: the actual length can be estimated by multiplying the apparent length by the factor that is appropriate for the zone that the filament lies in.

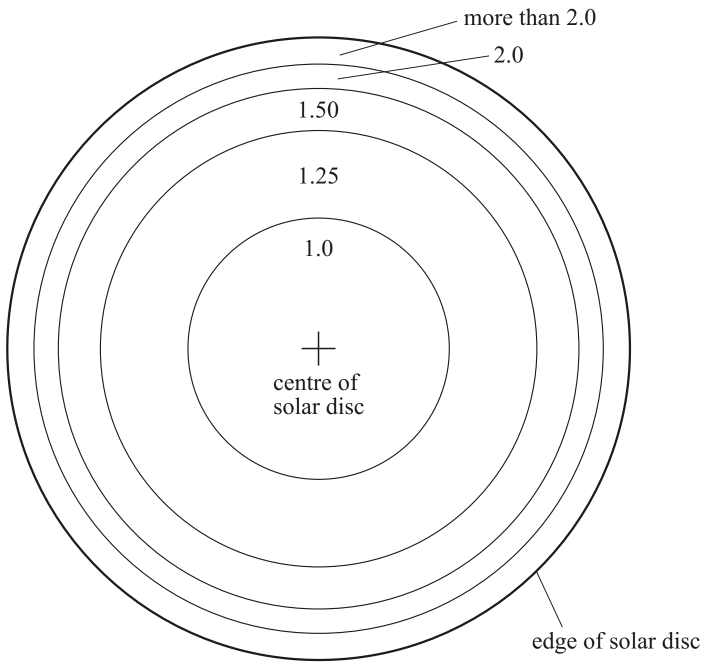

Figure 1 The factor by which the apparent length of any feature on the surface of the Sun should be multiplied to obtain its true length. This figure gives an estimate of this factor depending on where on the solar disc the feature is located. Note that this is only a very approximate correction factor – the true correction depends not only on the position of the filament on the solar disc but also on its orientation.

The major source of uncertainty in calculating the actual length of the filament is dominated by the uncertainty in the factor that is used to convert from apparent length to actual length. The uncertainty in an actual length that is calculated by multiplying the apparent length by the factor shown in Figure 1 is approximately as follows:

- Inner zone '1.0 zone' – relative uncertainty of about 5%
- '1.25 zone' – relative uncertainty of about 10%
- '1.50 zone' – relative uncertainty of about 25%
- '2.00 zone' – relative uncertainty of about 50%.

In the 'more than 2.00' zone, it is probably best not to use the factor as read from the diagram but to quote the apparent length as being a lower limit to the true length.

Question 5

(a) By reference to Figure 1, select the appropriate correction factor for each of your six chosen filaments.

(b) Complete your table of results for filaments A–F with their actual lengths and uncertainties.

We will now investigate the other reason why filament lengths might be underestimated –as you should have identified as an answer to Question 4. To find out whether filament A continues round the limb to the left we can note that the Sun rotates from left to right in these images. So you can look at the following day's image – and see whether more of filament A has appeared.

- Try this now – view the Hα image from 21/05/02. (Note that if you opened the 20/05/02 image in a new window then the 'back' button on the web browser won't take you back to the Image Archive. You should close the new window, and use the original window that displays the image and the thumbnails. Provided that you didn't close the original window it should still be present on the taskbar.)

Question 6

Using the same techniques as before, measure the new length of filament A.

- Now have a look at the next few days' images, up to 25/05/02, and observe the way that the large-scale features change from day to day.
- A good way to do this is to open two or three images in separate windows and then flick between them. Again, you can maximize the amount of screen space by right clicking on the thumbnail and then selecting the option to 'Open in New Window'.

Question 7

What do you notice about how the images change from day-to-day?

If the features evolve in this way, how long do they last? To answer this question we need to look at the images over a longer timescale.

- ■ If much of filament A lies at a latitude of about 30–40°, how long is its apparent period of rotation, i.e. how many days before 25/05/02 would we expect to see it in the same place, if it already existed then?
- ❑ From Figure 1.7 of *An Introduction to the Sun and Stars*, the intrinsic period of rotation at 35° is about 27 days. So the apparent period is about 29 days because the Earth has moved around the sun in its orbit during the 27 day period.

28 days before 25/05/02 is 27/04/02. Unfortunately there is no image for this date, but you can look at the image for the following day 28/04/02, 27 days before 25/05/02.

Question 8

Compare the image of 28/04/02 with that of 25/05/02. What do you notice?

Now go back another 27 days to the first available image – 01/04/02. Again, try opening the three images in three separate windows, so that you can flick between them.

Question 9

Compare the image of 01/04/02 with that of 25/05/02. What do you see now?

So from the limited data you have available you can't give a definitive answer to the question of how long the features last. But you have seen that although they

clearly evolve over a timescale of days and some filaments, for example, appear to last no more than a couple of weeks, some of the main features such as plages and major filaments have lifetimes that can be measured in months.

Having got this far, you may like to spend some more time tracking filament A as it evolves!

Answers to questions

Question 1

An Hα image is one where all the wavelengths of light (colours) have been filtered out except for a narrow band of red light around 656.3 nm, corresponding to the Hα line of the hydrogen spectrum. When we observe the Sun in ordinary white light – i.e. a mixture of all the visible wavelengths – the brilliant photosphere far outshines the far dimmer chromosphere overlying it. However the hydrogen atoms in the chromosphere are very efficient at emitting and absorbing photons from the transition between energy levels E_2 and E_3, which correspond to light of wavelength 656.3 nm. (See *An Introduction to the Sun and Stars*, Box 1.2). So hydrogen atoms in the chromosphere absorb most of the light that comes from the photosphere at this wavelength, and emit light of their own in at this wavelength. You can see an example of the reddish chromosphere emission in *An Introduction to the Sun and Stars*, Figure 1.17. Hence the detail we can see in an Hα image comes from the chromosphere rather than the bright photosphere.

Question 2

The image is the same brightness right across the disc; it doesn't show any limb darkening (Section 1.2.2, Figure 1.3). This is not an effect of the type of image – most of the light that is seen in an Hα image originates from the photosphere (remember that such light is absorbed by gas in the chromosphere) and hence such images should show limb darkening. However, the images have had their contrast enhanced, effectively taking out the limb darkening effect so it is easier to see the small-scale details.

Question 3

Four types of feature are clearly visible on the Hα image as shown in the sketch in Figure 2. These are filaments, prominences, plages and sunspots.

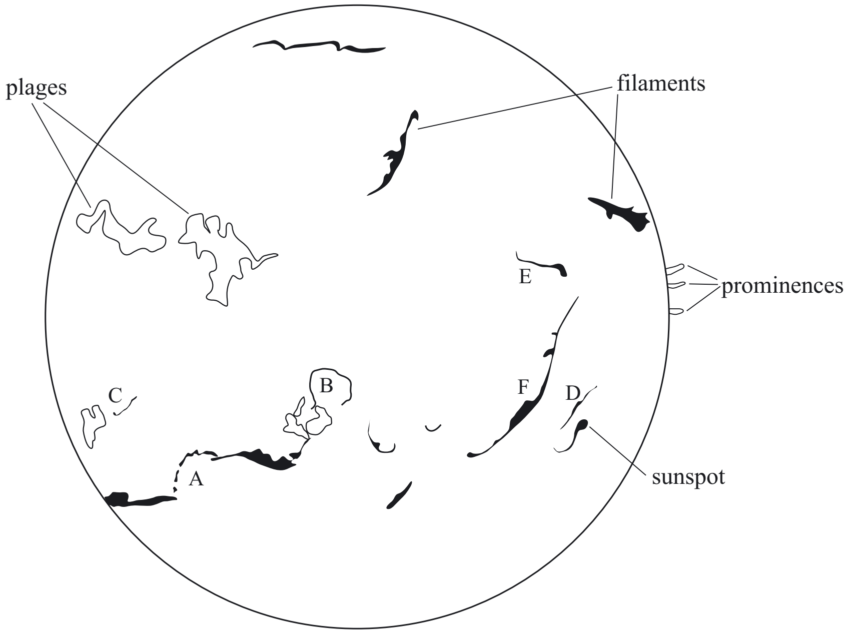

Figure 2 An example of a sketch of the features visible in the Hα image taken on 20/05/02. Note that the labels A–F denote individual filaments that are discussed in Question 4.

Notice the way that the long winding dark filament (call it filament A) across the left-hand half of the southern hemisphere turns into a prominence as you trace it leftwards towards the limb – towards the centre of the image you are looking straight down on it, but near the limb it becomes increasingly obvious that it has the form of clouds suspended over the photosphere.

You may have noticed that the small-scale features have changed subtly from the roughly circular convective granulations of the photosphere (Figures 1.11 and 1.12 of *An Introduction to the Sun and Stars*) to more elongated structures which show some sort of local alignment, in particular radiating out from sunspots. These features are governed by the local magnetic field, and are reminiscent of the sort of patterns iron filings make when dropped on to a sheet of paper with magnets underneath. The difference is because the density of the photosphere is high enough that mechanical energy determines its structure, whereas in the chromosphere the density drops off sharply (Figure 1.35) and the magnetic field dominates.

Question 4

(a) I have chosen the six filaments A–F shown in the sketch in Figure 2. You will probably have chosen some of the same – it doesn't matter if they are not identical.

The solar radius = $R_\odot = 6.96 \times 10^5$ km

So the length of filament A is given by

$(105 \text{ mm} \times 6.96 \times 10^5 \text{ km})/125 \text{ mm} = 5.85 \times 10^5$ km

The apparent lengths of all the filaments A–F are given in Table 1.

Table 1 Measured and calculated lengths of filaments for the 20/05/02 image.

Filament	Measured length on image/mm	Apparent length/km
A	105 ± 5	$(5.8 \pm 0.3) \times 10^5$
B	45 ± 5	$(2.5 \pm 0.3) \times 10^5$
C	15 ± 3	$(0.84 \pm 0.17) \times 10^5$
D	25 ± 3	$(1.4 \pm 0.2) \times 10^5$
E	40 ± 3	$(2.2 \pm 0.2) \times 10^5$
F	82 ± 5	$(4.6 \pm 0.3) \times 10^5$

(b) My shortest measured filament was C at 0.84×10^5 km, and the longest, A, was about 7 times longer at 5.85×10^5 km. I would be fairly confident that A is the longest filament on this image of the solar disc, but I think there could also be some shorter ones.

(c) There are two reasons why the actual lengths of filaments will be greater than the apparent lengths calculated here. The first is an effect that you met in Questions 1.2 and 1.10 of *An Introduction to the Sun and Stars* – that of foreshortening of features near the solar limb. The second reason applies only to filaments that extend to the limb – such as filament A here – we don't know from the image whether they continue round beyond the limb.

Question 5

(a) In most cases, the filament lies wholly or nearly wholly in a single zone on Figure 1. The corresponding correction factors are tabulated in Table 2. The most difficult case is filament A: this extends over several zones, and seems to disappear over the limb of the Sun. In this case, it is best not to try to adopt a correction factor, but to say that the apparent length is a *lower limit* to the actual length.

(b) The values of actual lengths are shown in Table 2. The uncertainties are derived from the dominant uncertainty – this is the uncertainty in the correction factor that is described in the text.

Table 2 The actual lengths of filaments A–F.

Filament	Correction factor	Relative uncertainty	Apparent length/km	Actual length/km
A	(see note)	–	5.8×10^5	$> 6 \times 10^5$
B	1.00	5%	$(2.5 \pm 0.3) \times 10^5$	$(2.5 \pm 0.3) \times 10^5$
C	1.50	25%	$(0.84 \pm 0.17) \times 10^5$	$(1.3 \pm 0.3) \times 10^5$
D	1.50	25%	$(1.4 \pm 0.2) \times 10^5$	$(2.1 \pm 0.5) \times 10^5$
E	1.25	10%	$(2.2 \pm 0.2) \times 10^5$	$(2.8 \pm 0.3) \times 10^5$
F	1.25	10%	$(4.6 \pm 0.3) \times 10^5$	$(5.8 \pm 0.6) \times 10^5$

Note: it is not possible to determine a correction factor for filament A, hence only a lower limit to the actual length is given.

Question 6

At first glance it certainly looks as if more of filament A has appeared. I get the same value for the diameter of the image – 125 mm – so the image is at the same scale, but now I measure the length of A as 120 mm. Hence my new estimate of the minimum length is $(120 \text{ mm} \times 6.96 \times 10^5 \text{ km})/125 \text{ mm} = 6.68 \times 10^5$ km, some 17 000 km longer than the previous day and now a length equivalent to nearly the solar radius. So our conclusion is that filaments range in length from a few thousand km to around half the solar diameter, or possibly even more. Remember that we have only studied one image in detail here.

Question 7

You can clearly see the large-scale features, i.e. the filaments/prominences, plages and major sunspots, moving to the right as the Sun rotates. As they move the shapes change subtly as parts of the filaments grow or shrink and the line of the filament moves about due to the shifting magnetic fields. This is partly due to the different angles of view as they move towards or away from the limb, and partly due to the actual evolution of the features.

Filament A seems to change between the 22 and 23 of May in that it is originally a continuous filament, but on 23 May it appears separated into two parts. On subsequent days the right hand part of the filament fades dramatically.

Question 8

The pattern of the main plages is very similar. As they are nearer to the solar equator than filament A, you would expect the period of rotation to be a little shorter, corresponding to using an image 27 rather than 28 days earlier. The pattern of the many of the smaller filaments is quite different, for example there are a pair of nearly vertical filaments just above and to the left of the most southerly of the main plage regions in the 28/04/02 image which seem to have completely disappeared by 25/05/02. However there was already a long filament in the region of filament A which, although it has changed shape, could be reasonably supposed to be the same structure. If you look closely you can see traces of this filament winding over towards the right-hand limb.

Question 9

Again, although the details change, some of the main features present on 28/04/02 clearly had their origins at least four weeks earlier. In particular, there was already a large filament in the southern hemisphere in just the position where we later find filament A.

Resources

The Big Bear Solar Observatory www.bbso.njit.edu

The Kanzelhöhe Observatory www.solobskh.ac.at/index_en.php

S282 Astronomy

Sunspot number

Study time: 90 minutes

Summary

In this activity you will plot and analyse sunspot data covering a 50-year period. You should have read to the end of Section 1.2 of *An Introduction to the Sun and Stars* before doing this activity.

This is the first activity that requires the use of a spreadsheet. Before starting this activity you should ensure that you have an appropriate spreadsheet package installed on your computer – see the *Using Spreadsheets* guide on the course website for more details.

If you are already familiar with using spreadsheets you will be able to complete the activity in a shorter time than suggested.

Learning outcomes

- Use a spreadsheet to perform simple calculations and to display data by plotting graphs.
- Present information in a spreadsheet effectively, accurately and in a way that makes sense.
- Analyse graphical data in both qualitative and quantitative terms.
- Develop your understanding of the solar cycle.

Background to the activity

Sunspots

The surface of the Sun contains a number of features including granulation, plages and sunspots. Sunspots can be seen as dark patches against the solar photosphere. This darkness is relative: sunspots have a temperature of around 4200 K compared with the hotter photosphere (6000 K) and thus appear dark against the brighter background.

Individual sunspots have a fairly short life span of just a few weeks, but the *numbers* of sunspots have been observed to fluctuate on a regular cycle. This variation is correlated with the cycle in solar activity.

Detailed records of sunspot numbers have been kept since 1849 but historical accounts go back much further and provide a long-running continuous record of solar activity.

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How sunspot numbers are computed today

In 1848 Rudolph Wolf (who went on to become director of the Zurich observatory) discovered the connection between the occurrence of sunspots and disturbances in the Earth's magnetic field. He also devised the modern convention for calculating a quantity that measures the number and extent of sunspots, the so-called *sunspot number*.

At times of high solar activity sunspots frequently occur in groups. To form the sunspot number the number of groups is multiplied by ten and added to the number of individual spots. Because the number of sunspots can vary as a result of the Sun's rotation and the location of observers, the overall figure is calculated as the average of all the counts made by a network of participating observatories. The resulting quantity is formally referred to as the Wolf number or Zurich number.

Records of the Wolf number have been kept since the mid-19th century. Using historical records, it has been possible to infer values of the Wolf number back to 1749.

WARNING

NEVER look at the Sun directly, not even with the naked eye and especially not through binoculars or a telescope even if fitted with filters. Permanent eye damage could result.

The activity

The aim of this activity is to produce a plot that shows how sunspot number has varied with time over the period 1953–2000, and to interpret this plot to estimate the period of the sunspot cycle. Much of the detailed description relates to the use of the spreadsheet package, but as you work through these notes you should bear in mind that the goal is to produce a chart that shows scientific information in a meaningful fashion.

The instructions given here assume that you will be using the StarOffice™ package that is supplied on the OU Online Applications CD-ROM. If you are already familiar with using another spreadsheet package (such as Microsoft Excel) you may want to use that to carry out the activity (we have supplied the required data file in Excel format). However, before starting you should be aware that these notes only give instructions on how to manipulate the StarOffice spreadsheet.

As you work through the examples feel free to experiment with the spreadsheet commands and menus. Don't worry if you make a mistake: in most cases you can use the key combination Ctrl-Z to undo the last action and bring you back to where you were before.

Before you start – create a folder for your work

In this activity you will be modifying a spreadsheet file and you will need to save this to your hard disk. Before you start it would be a good idea to create a folder in which you can store the results of your work.

Open the raw data file

The raw data for this activity is contained in a file called 'Sunspots_raw_data.sxc' (the Excel version of the file is called 'Sunspots_raw_data.xls').

- Start the S282 Multimedia guide program and open the folder called 'The Sun', then click on the icon for this activity ('Sunspot number').
- Press the Start button to access the folder on the DVD containing the StarOffice and Excel versions of the raw data file.
- Open the file you wish to use by double-clicking on it.

Save a copy of the file

Before you can make changes to the file you must save it to your hard disk.

- Use the File | Save as... menu command to save a copy of the spreadsheet into your work folder.

As you make changes to the spreadsheet you should save your work regularly to prevent any changes from being lost. From time to time make a *backup copy* of your work (using a different filename) in case you need to go back to an earlier stage. (If it all goes horribly wrong you can always go back to the original from the DVD!)

Formatting data

As the name implies, the spreadsheet you have just opened contains just the *raw* data of sunspot counts for each month over a period of nearly 50 years.

In order to turn this into a meaningful scientific document you will need to format this data, carry out some simple calculations on it, and plot a graph showing the variation in sunspot number over time. We will also review some of the important ideas about spreadsheets that are discussed in the *Using Spreadsheets* guide as we progress through this activity.

The spreadsheet is made up of a large number of cells arranged into *rows* and *columns*. Notice that along the top of spreadsheet the columns are labelled A, B, C, etc., and that down the left-hand edge of the spreadsheet, each row is numbered. Thus every cell on the sheet can be uniquely identified by its column label and its row number: this identification is called the *c*ell reference. For instance, the cell in the top left-hand corner of the sheet has a cell reference A1, the cell below it is A2 and the cell to the right of this is cell B2.

Each cell can contain either text or a number. Cells can also contain formulae that carry out calculations based on the contents of other cells. Normally a cell containing a formula will display the *result* of the calculation. Note that you can edit the formula itself in the input line – this is situated just above the top row of the sheet.

For more information, refer to the *Using Spreadsheets* guide.

Add titles

The raw data file that you have opened contains a list of years and months, and a column of numbers, but there is nothing to tell the reader what the data *are* or what they *mean*. (Don't forget that this may be yourself in a few months time, trying to remember what you did – so it pays to have everything clearly labelled and documented!). You are going to start by adding a heading and column titles to the spreadsheet.

Insert rows

To make room for the headings you need to create some blank lines above the data. At the left-hand side of each row there is a grey button labelled with the row number. Using the left mouse button, click on button 1 and then, holding down the left button, drag down to button 4, highlighting the first four rows.

Now right-click the mouse on any one of these four buttons. This will display a drop-down menu (Figure 1). Select Insert Rows from this menu to insert four blank rows. (You could also do this by selecting Insert | Rows from the main menu.) Notice that rows are inserted above the highlighted area and that the rows containing data are moved down.

Don't worry if it doesn't work exactly as you expected first time: remember, you can always use Ctrl-Z to undo the most recent change.

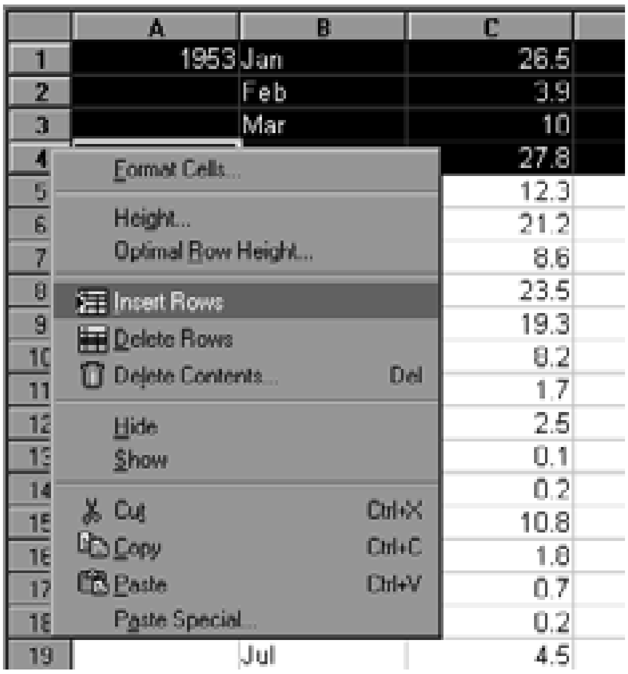


Figure 1 Inserting rows into the spreadsheet.

Enter spreadsheet titles

You are going to add a title in the top row of the spreadsheet.

- Select cell A1 by clicking it with the mouse. Notice that the cell is highlighted with a darker border, and the row '1' and column 'A' headings are also highlighted.
- Type in an appropriate heading, such as: 'Activity: Sunspot number'. As you type, notice that the text also appears in the input line directly above the top row of the spreadsheet. If you need to edit the contents of the cell at a later stage you can do so by selecting the cell and making the changes in this input line (alternatively you can double-click the cell and edit the contents directly). When you have finished typing press Enter (or click on another cell) to complete the entry.

  It is good practice to always record the source of any data that you work with. These data were obtained from the National Solar Observatory in the United States.
- Select cell A2 and insert an appropriate subtitle to describe the source of these data.

Format titles

The titles that we have added are OK, but the writing is quite small. We can improve the appearance of the spreadsheet by formatting the titles.

- Select cell A1 again. Increase the font size by selecting 20 from the Font Size box (located on the toolbar above columns A and B) (Figure 2). The whole of row 1 will increase in height. Similarly, increase the size of cell A2 to a font size of 14. This makes the headings easier to read.

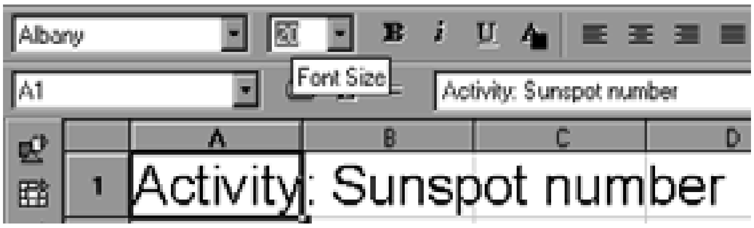


Figure 2 The font size box.

- Now select the whole of row 1 by clicking on the 1 button to the left of cell A1. Select Format | Cells ... from the main menu and choose a suitable colour from the Background tab. Do the same thing for row 2 (selecting a different colour if you wish).

Enter column headings

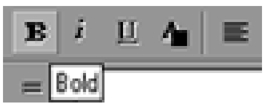
Now we need to include headings for each of the columns to explain what the data are.

- In cell A4 enter 'Year', in B4, 'Month' and in cell C4, 'Sunspot number'. You may have to increase the width of the sunspot number column – you can do this by selecting the cell C4 and then using Format | Column | Optimal width. Make sure the Default value option is not selected and then click on OK in the Optimal Column Width box that appears.
- Select all three cells (A4, B4 and C4) and select the Bold toolbar button (Figure 3a).

Format columns

The columns are now labelled, but the data do not line up very well with the headings: this will look very odd, especially when the sheet is printed.

- Centre the whole of column A by clicking on the A button at the top of the column and then using the Centred toolbar button (Figure 3b). Do the same with column B.

(a) 

(b) 

Figure 3 (a) The bold button and (b) the align centre button.

Column C contains numbers, and formatting these as centred probably isn't a good idea – in particular the decimal points won't line up! You can improve the appearance of the numerical data by telling StarOffice to display a fixed number of decimal places.

- Highlight column C and select Format | Cells ... On the Numbers tab, choose 1 decimal place (Figure 4).

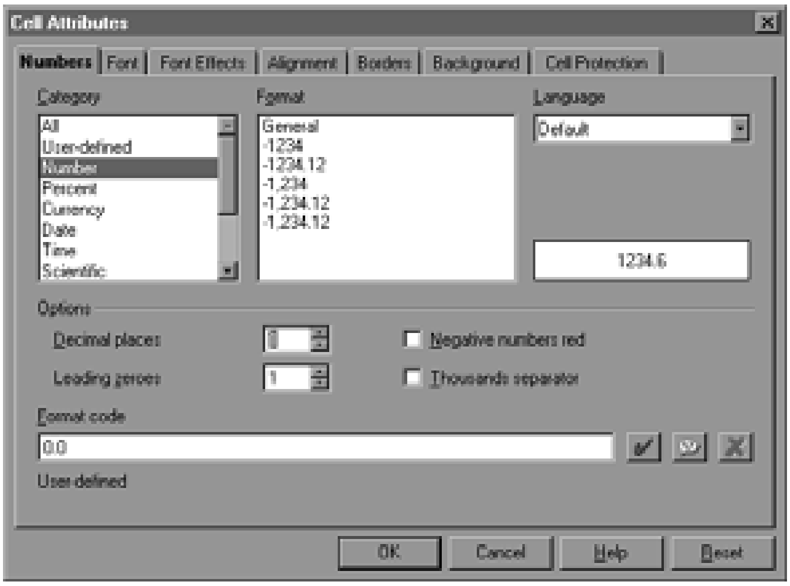


Figure 4 The cell attributes dialog box.

You may want to experiment with the formatting commands to present the data in a way that you like.

> Now would be a good time to *save* your work before going on to make more changes. As you continue to work through this activity remember to save your work regularly.

Calculations

(Refer to the *Using Spreadsheets* guide for further instructions on entering formulae.)

There are sunspot numbers for every month, but the Year column in your sheet only contains an entry in every twelfth cell. The Month columns contain data in the form of labels – that is, text rather than numbers.

In order to plot a graph of sunspot activity against the year we need a year value for each data point. A good way to do this is to combine the year and month to give a single numerical value (i.e. a decimal fraction of a year) that we can use for plotting.

First you need to make space to hold the new values by inserting a column.

- Highlight column C by clicking on the C button at the top. Then, keeping the mouse pointer over the C button, right-click and select Insert Columns. A new column C will be inserted, and the Sunspot number column moves to the right.
- Now go to cell C5 and enter the value '1953.0'. This is the starting point for the year calculations.
- In the cell beneath this enter the formula: '=C5+1/12' and press Enter. The result of this is the value 1953.08 and this should appear in the cell. (Notice that if you select the cell again, the formula itself appears in the input line and you can edit it there.)

The formula must now be replicated down the whole column, and this can be done by dragging.

- Select cell C6 (the one that contains the formula). Now click on the little black square in the bottom right-hand corner of the cell and drag down to row 580 (drag down past the lowest visible cell and keep the mouse button held down while the sheet scrolls past).

This will replicate the formula all the way down the column: each cell will calculate a value 1/12 of a year (i.e. one month) more than the cell above. You now have a column containing decimal year values, from 1953.0 (January 1953) to 2000.92 (December 2000).

You may want to format this column to show two decimal places so that the whole years line up with the fractional years. You'll also want to give the column a heading: 'Decimal year', for example. (If necessary you can adjust the column width by dragging the boundary between column headings.)

Create chart

(Refer to the *Using Spreadsheets* guide for further instructions on creating and formatting charts.)

The data in columns C and D can now be used to plot a chart of sunspot number against the year. Highlight both columns by clicking on cell C5 and dragging all the way down and across to cell D580. From the main menu select Insert | Chart ...

- A box labelled AutoFormat Chart will appear. You don't need to change anything here, so select Next >> to move onto the next step.
- From the Choose a chart type options select XY Chart (Figure 5a), followed by Next >> to move onto the next step.
- From the Choose a variant options, select Lines Only (Figure 5b), followed by Next >> to move onto the next step. (Note that while going through these steps you can always press Back to go to an earlier step if you need to correct something.)
- Finally, in AutoFormat Chart make the following changes:

  The tick-box next to Legend should be cleared (i.e. without a tick).

  Make sure the X Axis and Y axis tick-boxes are ticked – this will cause the axis titles to be displayed.

  Insert the text for titles as follows:

Chart title:	Sunspot number
Axis title / X Axis:	Year
Axis title / Y Axis:	Sunspot number

- Finally, press the Create button – this will draw the chart.

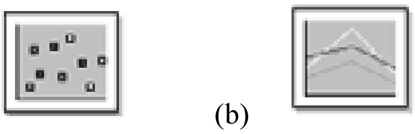
(a)  (b)

Figure 5 (a) The XY chart button and (b) the lines only button.

The chart will appear – usually at the top of the spreadsheet, so it is likely that you will have to scroll back up to the top of the sheet to see it.

Question 1

On the basis of the chart that you have just produced give a qualitative description of the behaviour of sunspot number over the period 1953–2000.

Question 2

Are there any aspects of the way in which the chart is displayed that could be improved to better show the behaviour of the sunspot number with time?

Formatting the chart

Having created the chart you will probably want to improve the way in which it displays these data, and maybe tidy it up to make it visually more appealing. This can be done by changing the *formatting*. In order to make changes the chart must be *selected*: there are two different levels of selection, signified by different borders around the chart:

- To resize, move or delete the chart: single click; the chart has green selection handles
- To edit or format the chart: double-click; the chart has grey border and small black squares
- To deselect the chart: click on any other cell in the spreadsheet.

Note that to swap between the 'resize, move, delete' selection and the 'edit, format' selection it is necessary to deselect the chart, before selecting it again.

The first thing that you are likely to want to do is to resize the chart to make it larger. To do this select it by single clicking and then dragging one of the green selection handles as required.

To change other aspects of the appearance of the chart you will need to make sure that the chart is selected for formatting (grey border and black squares).

There are many different things that you can do to change the appearance of your chart. Rather than describing all these the notes here give an example of how you could change one aspect of the chart – namely the appearance of the horizontal axis (X-axis). Other parts of the chart can be altered in similar ways and it is recommended that, if you have time, you should find out by experiment how to make other changes. (Always remember to save a copy of your file so that you can start again if the formatting ends up in a mess.)

If we wanted to make sure that the horizontal axes has tick marks for every year, and is labelled every ten years, we could make changes as follows:

- Select Format | Axis | X-Axis from the main menu.
- Select the Scale tab, then turn off the Automatic check boxes for Major interval and Minor interval.
- Set the value for the Major interval to 10: this will display a label every ten years.
- Set the value for the Minor interval to 1, then under the Tick-marks heading click on the tick-box labelled inner.

- While this box is open you can improve the display of numbers by clicking the Numbers tab, deselect the Source format option and then select 0 decimal places. This will make the years display as '1960' instead of '1960.0' and so on.
- Click on 'OK' when you are satisfied with the appearance of the axis.

You can change the formatting of just about any element of the graph by double-clicking on the part you want to change. As you move the mouse pointer over the chart a small label will appear telling you which item will be modified (you may need to be quite precise with the mouse to select the item you want!). In this way you can change the colour of the line plotted, add or remove background shading, gridlines and so on.

If you want to re-apply any of the chart type selections or titles choose Format | AutoFormat... from the main menu; this will take you through the AutoFormat Chart steps again.

Experiment with the various formatting options until you are happy with the appearance of your graph.

Printing the chart

Before printing your chart you should deselect it by clicking on any cell on the spreadsheet.

Experiment with the page layout to get the titles, chart and the first lines of data onto one page (you will probably want to print in landscape mode – to do this, select the menu item Format | Page then click on the Page tab, and then select the Landscape button).

Select File | Page Preview... from the main menu to preview exactly what will be printed.

You should aim to get a display similar to that shown in Figure 6 – you will probably need to reposition and resize the chart to make sure it fits onto the printed page.

Don't forget that the sheet contains over five hundred rows of sunspot data: be careful to select Page 1 only when printing – otherwise you may get 16 pages of printout!

Save your final spreadsheet so that you can refer to it again later.

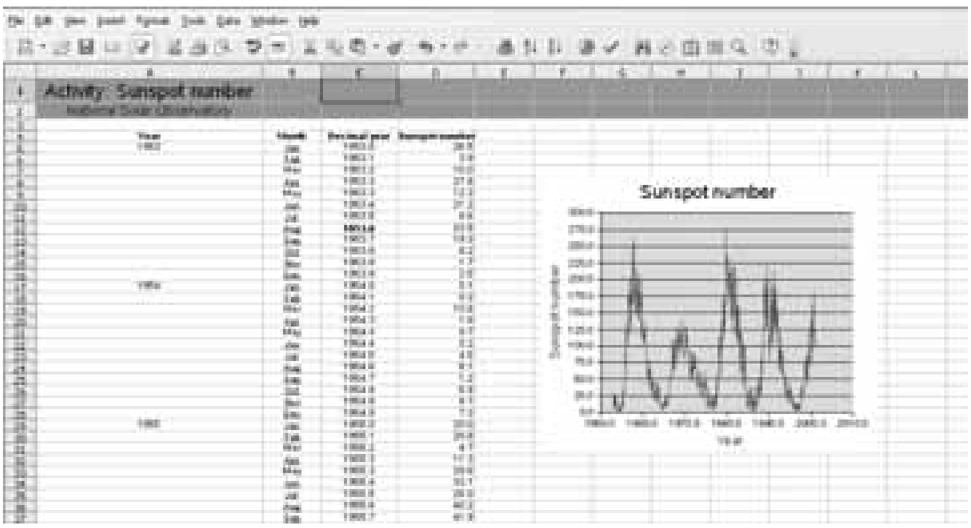


Figure 6 The sunspot number chart prior to printing.

The period of the sunspot cycle

You should now have a good representation of the sunspot number data for the period 1953–2000. The final part of this activity involves measuring the period of the sunspot cycle from the chart that you have produced.

Question 3

Give two different ways in which you could estimate the period of the sunspot cycle. For each method that you think of describe any advantages or disadvantages that it may have.

As described in the answer to Question 3, a good approach to measuring the period would be to measure the time between the first and last minimum of sunspot number and then to divide by the number of cycles. This is the approach that we will adopt here.

Question 4

(a) From your chart, estimate the date at which the first and last displayed solar minimum occurred.

(b) Hence calculate a value for the period of the solar cycle. (The calculation of the uncertainty in this result is described in the Question 4 answer below.)

Answers to questions

Question 1

The chart for 1953–2000 shows that the sunspot number changes with an approximately periodic variability. Within each cycle the sunspot number varies from a minimum to a maximum and then returns to a minimum again over a period of about ten years or so. However the variation does not repeat perfectly from cycle to cycle, and it is clear that the maximum value of sunspot number differs between cycles. The sunspot number also shows erratic variation, which seems to occur on a much shorter timescale than the period of the cycle.

Question 2

The answer to this question is somewhat subjective, but it is likely that the chart that is initially produced is rather small. Furthermore the default chart does not make best use of the available drawing area as it tends to extend the horizontal axis to a value that is substantially beyond the last data point. (Details on how to change these aspects of the chart are described in the notes after Question 2.)

Question 3

There are several ways of estimating the period from data such as those shown in the chart. Rather than describe all options in detail, here we outline the general principles behind the measurement of the period. You should be able to relate your two methods to these general approaches.

It is necessary to measure the time interval between repeating features such as the maxima or minima of sunspot number. However, the choice of feature will have an effect on the precision of the measurement. It is clear from the chart that the

time of sunspot maximum is more poorly defined than the time of sunspot minimum. Thus measurements based on estimating the times of the minima are likely to have smaller uncertainties than those based on the maxima.

The second point to consider is the number of intervals that are used in the measurement. One approach could be to measure the time taken for one cycle to repeat – so for instance you could measure the time interval between the first and second minimum. This has the disadvantage that it only uses a small amount of the information provided. This approach could be extended by finding the average interval between all successive cycles (i.e. between first and second, second and third, third and fourth). However the problem with measuring over a single cycle is that any uncertainty in determining the time of the minimum is a substantial fraction of the time being measured. This uncertainty can be reduced by measuring the time taken for several cycles to repeat, then dividing by the number of cycles.

Question 4

(a) The data at the earliest dates shown on the chart (around 1953) are close to a minimum, but the coverage of the minimum is not sufficient to enable a reasonable estimate to be made of when it occurred. The first minimum that is seen in its entirety is the one around 1964, and it is better to use this as the first minimum. The date of the last minimum appears to be about 1996.

(b) Since there are three cycles between 1964 and 1996, the period is given by

$P = (1996 - 1964)/3 = 10.7$ years

So the period of the sunspot number as determined from these data would be 10.7 years.

The uncertainty in the period that you determined in Question 4 can be found as follows. If a quantity z is found by subtracting one number y from another x (i.e. $z = x - y$), then the uncertainty in z is related to the uncertainties in x and y by

$$\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$$

(Note that the symbol Δ is used to indicate the uncertainty in a quantity, so, for instance, Δx is the uncertainty in x.)

For the sake of this example we will use the following values:

- Time of first minimum: 1964 (note the minimum around 1954 cannot be established from these data as the coverage only starts in 1953).
- Time of last minimum: 1996
- Number of cycles between these minima: 3

This yields a value for the period P of 10.7 years (see the answer to Question 4).

The time of any minimum can be estimated to within ±2 years, so the uncertainty in the time interval between the first and last minimum is

$$\Delta t = \sqrt{2^2 + 2^2} = \sqrt{8} = 2.8 \text{ years}$$

This represents the uncertainty in the time for three complete cycles, so the uncertainty in the period is one-third of this value

$\Delta P = (2.8 \text{ years})/3 = 0.9$ years

So the uncertainty in the period that would be obtained from the analysis of the chart data would be ±0.9 years, and the final result can be quoted is that these data yield a value for the period of (10.7 ± 0.9) years.

S282 Astronomy

The magnetic Sun

Study time: 20 minutes

Summary

In this activity you will view a video clip about solar granulation and magnetic features of the Sun such as sunspots and solar flares. The sequence shows observations that illustrate how these phenomena evolve with time. This sequence is best watched after you have read Section 2.3 of *An Introduction to the Sun and Stars*.

Learning outcomes

- Recognize the observed characteristics of the following solar phenomena: solar granulation; sunspots; solar flares; filaments.
- Appreciate that sunspots and solar flares are primarily magnetic phenomena.

The activity

- To find the video clip for this activity, start the S282 Multimedia guide and then click on The magnetic Sun in the folder called 'The Sun' in the left-hand panel.
- Press the Start button to run the video sequence.

Note that at time code 07:41 a schematic diagram of a flare appears on screen that the narration says is reproduced in these notes. This diagram is not given here, but you should instead look at the very similar diagram in Figure 2.25 of *An Introduction to the Sun and Stars*.

After you have watched the video clip, answer the following question.

Question 1

Watch the final 15 seconds of the video clip again (starting from the on-screen clock reading of 08:31). As you do so, note the clock readings when the following solar phenomena are visible:

- sunspots
- flare
- filaments.

Notes

The video sequence can be summarized as follows (the timings are those that appear on screen as the various topics begin).

00:00 You first saw the features that are *not* due to magnetic activity – granulation due to convection cells, each some 1000 km across covering the photosphere. The granulation also participates in larger-scale ordered motion. This is called supergranulation.

00:56 You then saw features that *are* due to magnetic activity, including sunspots, prominences and flares.

01:32 Sunspots, and the sunspot cycle, were explored in some detail, including the dynamic interactions with the plasma flow and magnetic fields. The sunspot cycle has an 11-year period in terms of numbers of sunspots and their position on the photosphere, but a 22-year cycle if magnetic polarity is taken into account.

05:50 Flares – regions showing a sudden release of energy across the electromagnetic spectrum – were shown to be associated with sunspots, and characterized by particles flowing down magnetic arches towards the photosphere.

07:41 See the comment at the start of these notes about the schematic diagram of the flare.

08:20 You saw a sequence of observations showing various solar phenomena.

08:31 The narrator asks you to identify solar phenomena that are visible (Question 1).

08:45 Sequence ends.

Video credits

Narrator – Alan Cooper (The Open University)

Producer – Tony Jolly (BBC)

Answer to Question 1

Here are the clock readings for the solar phenomena you were asked to identify:

Phenomenon	Clock reading (at first appearance of phenomenon)
sunspots	08:31
flare	08:33
filaments	08:39

S282 Astronomy

Our invisible Sun

Study time: 40 minutes

Summary

In this activity you will view a video sequence that describes how our understanding of solar phenomena has been advanced by making observations across the electromagnetic spectrum, from radio to X-rays. The video sequence refers to concepts relating to electromagnetic radiation that were introduced in Chapter 1 of *An Introduction to the Sun and Stars*, but is primarily concerned with the observation of phenomena that are related to solar activity. This sequence is best watched after you have read to the end of Chapter 2 of *An Introduction to the Sun and Stars*.

Learning outcomes

- Appreciate the importance of multi-wavelength observations in solar astronomy and in astronomy in general.
- Recognize some of the key characteristics of solar flares and coronal mass ejections.

The activity

This activity is based around a long (about 20 minute) video sequence, which was originally shot in 1993. Although there have been advances in observing technology since the video was made the principles behind the observations are still relevant today.

- Start the S282 Multimedia guide and then click on Our invisible Sun under 'The Sun' folder in the left-hand panel.
- Press the Start button to run the video sequence.

After you have watched the video sequence read the summary provided in the notes below.

Notes

The video sequence can be summarized as follows.

The sequence begins with a sunrise as seen in X-rays by the satellite Yohkoh. After glimpses of images in ultraviolet radiation it turns to visible images, in particular those made at the Big Bear Solar Observatory (BBSO).

Rather than making brief specialized observations the BBSO has monitored the Sun over many years. Harold Zirin, its founder and director and famous solar astronomer, explains and demonstrates the functions of its three telescopes and detectors, all on the same mounting. The largest has a 26 inch diameter mirror in

an evacuated housing, and was originally made for launch as part of a satellite-borne solar telescope. In fact, very long-term monitoring is of necessity ground-based and therefore confined to the visible and near infrared regions. Despite this restriction a vast amount of experience of the variety and timescales of solar active regions has been gained. In conjunction with a sister observatory in Beijing, China, round-the-clock operation can be achieved. Observations at other wavelengths are correlated with this nearly continuous record.

One of the advantages of the great intensity of light from the Sun is that one can afford to use filters with a very narrow acceptance of wavelengths. Narrow filters are able to isolate the spectral lines of particular chemical elements. The video shows this in action using observations made at the wavelength of the hydrogen alpha (Hα) line. Images at this wavelength reveal phenomena taking place above the photosphere in the solar atmosphere, including motions (via Doppler shifts) and details of the magnetic field strength. The various images reveal a wealth of phenomena associated with flares, sunspots, and other sorts of activity, in all of which the gas (plasma) in the solar atmosphere interacts with the magnetic field.

To investigate flares further use is made of images at radio wavelengths, as is demonstrated by Mukul Kundu, using images from the Very Large Array in New Mexico. By combining the radio and visible images in a flare region a clear picture is obtained of the way in which a radio flare is generated by electrons spiralling in a magnetic field that arches up from the Sun. The advantage of combining images from different wavelengths was very obvious – this advantage applies throughout astronomy.

The solar atmosphere above the photosphere (i.e. towards the corona) is well seen in ultraviolet radiation. Not only is the temperature different, but, as Ken Phillips showed, the structure is different, showing a type of granulation, but at scales very different from that in the photosphere. Looped structures break through these layers and surge upward towards the corona. The most energetic events are seen in X-ray images.

One effect of the plasma surges may be coronal mass ejections: the ejection of thousands of millions of tons of matter out into the solar wind, and in some cases towards the Earth. Richard Harrison showed how the stream of plasma can be followed by careful recording of radio scintillations, which indirectly yields images of the plasma. If it strikes the top of the Earth's atmosphere, auroral displays are produced. Beyond the Earth the plasma travels on out towards the heliopause, the magnetic boundary between the Solar System and the interstellar medium.

The fundamental nature of flare activity is underlined by the fact that it occurs in other stars. Stuart Bowyer explains a particularly revealing observation of a flare star (AU Mic) by the satellite-borne Extreme Ultraviolet Explorer (EUVE) telescope. We can detect only very large flares in other stars because they are vastly more distant than the Sun. But their existence in turn raises the question of whether such huge flares could ever occur on our Sun. If they did, the huge increase of radiation would probably be fatal for most life-forms on Earth. This raises the art of flare prediction above the level of academic interest!

Video credits

Speakers (in order of appearance)

Harold Zirin (Big Bear Solar Observatory)

Mukul Kundu (University of Maryland)

Ken Phillips (Space Science Group, Rutherford Appleton Laboratory)

Richard Harrison (Space Science Group, Rutherford Appleton Laboratory)

Stuart Bowyer (University of California at Berkeley)

Producer – Tony Jolly (BBC)

Narrator – Veronika Hykes

Course team consultants – Alan Cooper and Bob Lambourne

S282 Astronomy

Stellar distance and motion

Study time: 90 minutes

Summary

In this exercise you will be using measurements of parallax and proper motion from the European Space Agency (ESA) Hipparcos satellite to calculate the distance and speed of a number of stars along with the uncertainties in these quantities. You will construct a spreadsheet to carry out the necessary calculations. This activity is related to material in Sections 3.2.1 and 3.2.2 of *An Introduction to the Sun and Stars*.

Learning outcomes

- Calculate the distance and transverse speed from measured parallax and proper motion.
- Understand the concepts of absolute and relative uncertainties and the ability to determine them in derived quantities.
- Use a spreadsheet to perform calculations on astronomical data.

Background to the activity

Although the familiar pattern of the constellations appears unchanging, the stars are not stationary in space. On a large scale stars move with the rotation of the galaxy, but on a local scale stars are also moving through space with respect to the Sun. Over a long period of time nearby stars will appear to move compared to the background of distant stars. Figure 1 (*overleaf*) shows the appearance of the constellation Ursa Major over a period of time. This movement is known as *proper motion* and is measured in arc seconds per year.

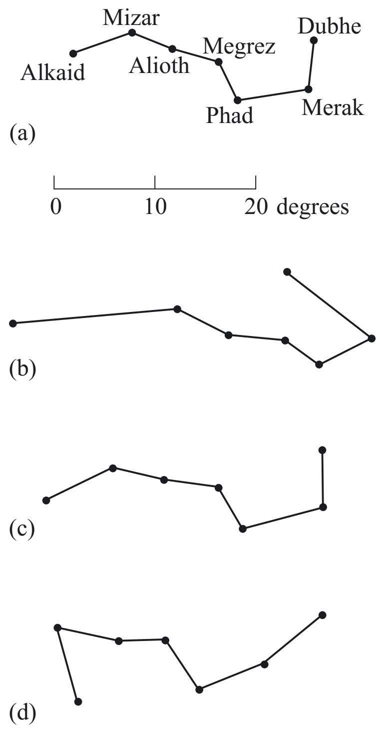


Figure 1 (a) The Plough; part of a larger constellation called The Great Bear (Ursa Major) as it appears today, and (b) 100 000 years ago, (c) 5000 years ago, and (d) 100 000 years in the future. (This is Figure 3.2 in *An Introduction to the Sun and Stars*.)

Of course, a star will only appear to move if it has a component of motion across our line of sight: a star that was moving directly towards or away from the Sun would not appear to move. In general, the actual movement through space is at an angle to our line of sight, giving a combination of transverse and radial motion as shown in Figure 2, so the total motion through space depends on both the transverse and radial speeds.

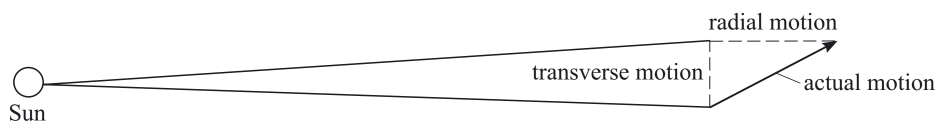


Figure 2 The actual motion of a star through space is a combination of the transverse motion and radial motion.

Typical speeds of motion can range up to over a hundred kilometres per second.

It is not possible to measure this motion directly – as with so many things in astronomy it is necessary to piece together information from a number of different sources. In this case these sources are:

- motion against background stars (proper motion)
- parallax measurements to get distance
- measurement of Doppler shift to give radial speeds.

Measurements

As mentioned above, the movement of stars against the background of distant stars is known as proper motion. It is an angular measurement: the actual speed of the motion through space will depend also on the distance of the star from the solar system. A fast moving star that is far from the Sun will show only a small proper motion, whereas a closer star moving at the same speed would cover a larger angle of view in the same time and therefore have a larger proper motion.

As discussed in *An Introduction to the Sun and Stars* Section 3.2.1, the distance to nearby stars can be measured using the method of parallax. As the Earth orbits the Sun the changing point of view causes nearby stars to appear to move across the sky relative to more distant stars in the background. Over the course of a year the star will appear to describe a parallactic ellipse. Given the fixed baseline of the Earth's orbit the size of this parallax movement depends only on the distance of the star: a more distant star will show a smaller parallax movement than one nearby.

The most convenient unit of measurement for stellar distances is the parsec. (See Figure 3.7 in *An Introduction to the Sun and Stars*.)

For a star with a parallax of p arc seconds the distance (in parsecs) is given by:

$$d/\mathrm{pc} = \frac{1}{p/\mathrm{arcsec}}$$ (*An Introduction to the Sun and Stars*, Equation 3.7)

By combining the distance measured by parallax and the (angular) proper motion the actual transverse velocity can be found.

In addition to moving across our line of sight (transverse motion) the intrinsic velocity of a star may also have a component towards or away from the Earth (radial motion). Motion along a line of sight cannot be detected as movement against the background stars, but it can be measured instead by observing the Doppler shift of known lines in the star's spectrum.

In this activity you will use data from the Hipparcos mission, which measured the positions of many stars without the blurring introduced by the presence of the Earth's atmosphere. Since Hipparcos was designed to measure positions of stars and not their spectra it provided parallax and proper motions but not radial velocities. You will use these data to investigate the distances and transverse motions of a few of these stars.

The data

Table 1 contains data for a selection of nearby stars obtained from the ESA Hipparcos catalogue (http://www.rssd.esa.int/Hipparcos/).

The uncertainties in the Hipparcos data can be regarded as ±1 milli-arcsec for the parallax and ±1 milli-arcsec yr^{-1} for the proper motion.

Table 1 Stellar motion data for a selection of nearby stars.

Star name	Parallax /milli-arcsec	Proper motion /milli-arcsec yr^{-1}
Alpha CMa (Sirius)	379.2	1339.4
Alpha Ori (Betelgeuse)	7.6	29.4
Beta Ori (Rigel)	4.2	2.0
Alpha Tau (Aldebaran)	50.1	199.5
Alpha UMa (Dubhe)	26.4	140.9
Kapteyn's star	255.3	8670.5

The activity

The aim of the activity is to calculate the distance and transverse speed of all seven stars using the data supplied. Furthermore you will also calculate the uncertainties in these two quantities. Of course you could do all the required calculations using a calculator: a reasonable prospect for data from one star, but less appealing for a larger data set. This is why a spreadsheet is used here – it provides a way of repeating the same type of calculation many times over. Although you might have to spend some time learning how to use a spreadsheet, by the end of this activity you should be able to appreciate its potential for helping you to carry out calculations. The *Using Spreadsheets* guide on the course website gives additional help.

Set up spreadsheet

Create a new sheet

If you have not already done so, start StarOffice™. The first thing to do is to create a new spreadsheet to hold the data and calculations.

From the main StarOffice menu select File | New | Spreadsheet. This creates a new, blank sheet.

Data (parallax and proper motion) for seven stars are given in Table 1. We want to calculate the distance and proper motion for each of the stars along with the uncertainties in both these quantities. A sensible way to set out the spreadsheet would be in seven columns, headed: 'Star name', 'Parallax', 'Distance', 'Uncertainty in distance', 'Proper motion', 'Transverse speed', 'Uncertainty in transverse speed'.

Enter data

Now enter the data given in Table 1 into the appropriate columns in the sheet, leaving blank columns for the results. Be sure to label the rows and columns clearly. In particular spreadsheets are good at manipulating numbers, but they do not keep track of units very well so make sure that you have included the units in the column heading labels.

Save sheet

Once you have started entering data it is a good idea to save the spreadsheet. Since this is a new spreadsheet you need to choose a name and, most importantly, a location for the spreadsheet file.

(If you have not already done so, it is a good idea to create a separate folder to hold all of your S282 work, to make it easier to find later on.

From the StarOffice menu, select File | Save as.... Enter a meaningful name for your sheet, and select the folder where you want to store the document. Press Save when you are ready.

As you continue to work through this activity, remember to save your work regularly.

Formatting and labelling

As in the earlier activities, you should consider carefully the layout of your spreadsheet. In addition to the data you should include sufficient labelling and other information to allow the user of the spreadsheet to understand what is going on. (You may wish to refer back to the formatting that you did if you completed the earlier activity on sunspot numbers.)

As a minimum, you should include the following information on your sheet:

- title
- reference to source of data (including website address (URL) if appropriate)
- explanatory comments and notes.

You might also want to consider using shading to indicate which cells contain input data and which ones contain the results of calculations (for example, cells containing data and constants could have a light yellow background, and those containing results of calculations a darker yellow). However, don't make it too lurid and give some thought to what the sheet might look like if printed out in monochrome.

Feel free to experiment with different formats, but try to maintain a consistent style as you create more sheets.

At this stage you should have a sheet that resembles the one shown in Figure 3. (It may be useful to ensure that your sheet uses the same row numbers and column letters as the example shown in Figure 3, since the explanatory notes below use cell references that relate to this spreadsheet.)

	A	B	C	D	E	F	G
1	**S282 Activity – Stellar distance and motion**						
2	**Author:** my name	**Last updated:** today's date					
3	**Source of data: http://astro.estec.esa.nl/Hipparcos**						
4							
5	parsec	3.09E+013 km					
6	milli-arcsec yr^-1	1.54E-016 rad s^-1					
7							
8	**Star name**	**Parallax**	**Distance**	+/-	**Proper motion**	**Transverse speed**	+/-
9		milli-arcsec	parsec		milli-arcsec yr^-1	km s^-1	
10	Alpha CMa (Sirius)	379.2			1339.4		
11	Alpha Ori (Betelgeuse)	7.6			29.4		
12	Beta Ori (Rigel)	4.2			2		
13	Alpha Tau (Aldebaran)	50.1			199.5		
14	Alpha UMa (Dubhe)	26.4			140.9		
15	Kapteyn's star	255.3			8670.5		
16							

Figure 3 The spreadsheet prior to carrying out any calculations.

If you are not familiar with spreadsheet formatting you may find difficulty in making an exact copy of the spreadsheet below because StarOffice re-formats your typing. If you want to learn how to turn off this reformatting see Appendix A, at the end of these instructions.

Calculate distance

Once the data have been entered you need to enter a suitable formula to calculate the distance. You should decide for yourself what formula to enter. (Think about what calculation you need to do based on Equation 3.7 given above.)

Units

As mentioned earlier, spreadsheets do not know about units. You must therefore think very carefully about units and account for them yourself in any formulae you enter.

■ Here, the parallax values are given in milli-arcsec. What must you do in your formula to account for this?

❑ You must explicitly divide the parallax values by 1000 to convert them from milli-arcsec to arc seconds. If you use a value more than once in a formula, you must convert the units every time they are used.

Formulae in a spreadsheet always start with an '=' sign. Click on the first blank cell in the Distance column of your table (in the spreadsheet shown in Figure 3, this is cell C10), and type '=' followed by the formula you have chosen.

(Consult the *Using Spreadsheets* guide for assistance on entering formulae.)

Checking

This is *very* important: as you have probably just found out, entering formulae can be a little tricky at first. You should *always* check the results of your calculations before proceeding.

How could you check that the formula you have just entered is correct? Well, one way would be to do the calculation by hand or on your calculator to check the result. Another method is to try it out on some data where you know the answer already. For example, a parallax of 1000 milli-arcsec should give a result of 1 parsec, and 100 milli-arcsec should give 10 parsecs (always check at least a couple of numbers to make sure you haven't got the right answer by accident!). You could also look up the actual known distance to one of the stars in Table 1 and check your calculation that way.

If you are having difficulty in entering a correct formula, see Note 1 in the 'Notes' section at the end of these instructions.

Once you are happy that your calculation is working correctly you can replicate it down the whole column by dragging the small black handle at the lower right-hand corner of the cell containing the formula.

Calculate uncertainty in distance

The calculations of distances are based on parallax data that are known to an accuracy of ±1 milli-arcsec. Having calculated the distances it is now necessary to calculate the *uncertainty*.

How is the uncertainty in the calculated distance related to the ±1 milli-arcsec uncertainty in the parallax measurements? For this calculation the appropriate rule is that for a quantity x, the *relative* uncertainty in $y = 1/x$ is the same as the relative uncertainty in x (so if x is accurate to 10%, y is also accurate to 10%). (See Box 1.)

Box 1 Relative and absolute uncertainties

Whenever a measured quantity is reported it is very important to give an indication of the *accuracy* of the result. This is done by giving both the measurement and the uncertainty in the measurement.

If a quantity A has been measured to an accuracy of ΔA, then the result would be quoted as: $A \pm \Delta A$. The quantity ΔA is known as the *absolute uncertainty*.

The absolute uncertainty does not tell the whole story, however. Clearly a value of 100 ± 1 represents a higher accuracy than a value of 10 ± 1. It is therefore often useful to express the uncertainty as a *relative* quantity – for a measurement of $A \pm \Delta A$ the *relative uncertainty* is the fraction $\Delta A/A$.

Relative uncertainties are frequently expressed as *percentages*, so for a quantity ($A \pm \Delta A$) of 10 ± 1, the relative uncertainty $\Delta A/A$ would be 1/10, or 10%.

In this case, the distance d is calculated as the reciprocal of the parallax p.

The *relative uncertainty* in d is thus the same as the relative uncertainty in p:

$$\frac{\Delta d}{d} = \frac{\Delta p}{p}$$

In order to calculate the *absolute* uncertainty Δd, this equation can be rearranged as:

$$\Delta d = d\frac{\Delta p}{p}$$

giving a simple formula that can be used in the spreadsheet.

Notice that doing it this way saves a lot of unnecessary arithmetic: p and Δp can be left in units of milli-arcsec, and there is no need to convert $\Delta p/p$ into a percentage and then back again.

The formula gives Δd directly. You should always try to reduce your calculations to a simple formula before starting to work with the actual numbers.

Click on the cell that will hold the result of the calculation of uncertainty in distance of the first star in the list, (in Figure 3 this is cell D10).

Now enter the appropriate formula for the distance uncertainty calculation. If you are having difficulties in doing this see Note 2 in the 'Notes' section found towards the end of this activity.

Check this formula on the data for the first star before replicating it down the column.

Question 1

What do you notice about the uncertainties in the distances? What implications does this have for using the parallax method of determining stellar distances?

At first sight, it might appear that all that is needed to extend the range of the method is to increase the accuracy of the angular measurements of parallax. However, because of atmospheric blur ('seeing'), stars appear as a blur of finite size rather than an absolute point if measurements are conducted from the ground. Also, the actual diameter of the star may be larger than the parallax movement we are trying to measure (Betelgeuse has a diameter of 50 milli-arcsec). As you can imagine, it is very difficult to measure movements very much smaller than the diameter of the star's image. Even for telescopes mounted on spacecraft, like Hipparcos, there is a theoretical limit to the size of an image that is seen for a point-like source, which is related to the diameter of the telescope; the larger the telescope, the smaller the image that can, in theory, be produced. This theoretical limit will be reached only if the surfaces of the mirrors and/or lenses in the telescope are accurate to a fraction of the size of a wavelength of light and the stability of the pointing of the telescope is precise. For Hipparcos this theoretical limit to the image size is around 400 milli-arcsec, so the accuracy to which the position of the centre of such a star's image can be determined is quite remarkable. It is equivalent to the width of a golf ball viewed across the Atlantic Ocean!

Calculate transverse speed

In order to calculate the transverse speed from the distance and proper motion a little geometry is required:

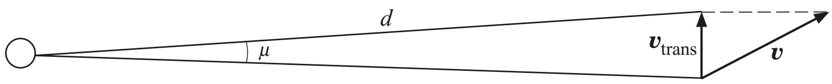


Figure 4 Geometry of stellar motion.

Since the angle μ is very small, the formula is apparently quite straightforward:

$$v_{\text{trans}} = d\mu$$

But once again you will need to think carefully about *units*.

■ The values that you have for d and μ so far are given in parsecs and milli-arcsec per year respectively. If you want to obtain an answer for v_{trans} in km s^{-1}, what units for d and μ do you need to use in the formula $v_{\text{trans}} = d\mu$? (You may want to refer to *An Introduction to the Sun and Stars* Section 3.2.1).

❑ For the calculation to give a result in km s^{-1}, the distance d must be expressed in km and the proper motion μ in radians s^{-1}. You will therefore need to *convert* the values into the correct units. (See Box 2.)

Box 2 Unit conversions

The following conversion factors are useful for the calculations required in this activity

1 radian = 180/π degrees = (180/π) × 3600 arcsec = 206 265 arcsec
= 2.06×10^8 milli-arcsec

1 milli-arcsec = $1/(2.06 \times 10^8) = 4.8 \times 10^{-9}$ radians

1 year = 365.25 days = 365.25 × 24 hours = 365.25 × 24 × 3 600 s
= 3.16×10^7 s

1 milli-arcsec $yr^{-1} = (4.8 \times 10^{-9}$ radians$)/(3.16 \times 10^7$ seconds$)$
= 1.54×10^{-16} radians s^{-1}

1 parsec = 3.09×10^{16} m = 3.09×10^{13} km

Question 2

The spreadsheet gives distance in parsecs and proper motion in terms of milli-arcsec per year. What are the relevant conversion factors to (a) express the distance in kilometres, and (b) express the proper motion in rad s^{-1}?

You now need to enter a formula for the transverse speed of the first star in the list (in Figure 3, this is cell F10). Take care to include the conversion factors discussed in Question 2. You may find it useful to know that numbers in scientific notation can be input into a spreadsheet as shown in the examples in Table 2.

Table 2 Examples of how numbers in scientific notation are represented in a spreadsheet.

Number	Spreadsheet
3.84×10^{26}	3.84E+26
1.602×10^{-19}	1.602E-19

As in the previous calculations, you should check your result before replicating the formula for all seven stars. If you are having difficulties in obtaining or entering the correct formula see Note 3 in the 'Notes' section.

Appendix B to this activity shows how you can use named references to help clarify the formulae that you use in spreadsheet cells. This technique can be used in the formula that you have used for the transverse speed, so if you would like to try out using this facility it is recommended that you read Appendix B now. If, however, you are new to using spreadsheets it is probably best to skip Appendix B on your first reading of this activity.

Again, it is good practice to save the spreadsheet at this point.

Calculate uncertainty in transverse speed

The remaining quantity to be calculated is the uncertainty in the transverse speed. This is a bit more involved than the calculations that you have already done, and

this might be an appropriate place to take a break from this activity. If you do, make sure to save your spreadsheet before closing the StarOffice program.

The transverse speed is found by multiplying together two quantities, d and μ which both have associated uncertainties. In the following section (and Box 3), we'll look at how we handle uncertainties that are formed by the combination of two quantities.

Box 3 Combining uncertainties

When two quantities are added or subtracted the absolute uncertainties must be combined. This is not as simple as adding or subtracting the uncertainties.

Adding or subtracting quantities:

If you have two quantities $(x \pm \Delta x)$ and $(y \pm \Delta y)$ then the absolute uncertainty in $(x + y)$ is given by: $\sqrt{\Delta x^2 + \Delta y^2}$. The uncertainty in $(x - y)$ is the same, so be careful when subtracting quantities!

This process of squaring and adding is known as combining the uncertainties *in quadrature*. It gives a smaller result than simply adding the uncertainties, and thus takes into account the random nature of uncertainties in the two quantities x and y, which are independent of each other.

A simple way of understanding this is to consider an example: x and y are measurements of the length of two blocks of wood using a mm ruler. Each has an *estimated* uncertainty, Δx and Δy respectively. What this is really saying is that the true answer for the length of x is *likely* to lie in the range from $(x - \Delta x)$ to $(x + \Delta x)$. The measured value may be smaller or larger than the true value and the probability of it being Δx or more from the true value is small. Likewise, for the length y and its estimated uncertainty Δy. If we add these quantities (i.e. $z = x + y$) then we have the combined length of both blocks. However, it is *very* unlikely that the measured values of x and y were *both* too large (or both too small) by amounts Δx and Δy, as implied by assuming $\Delta z = \Delta x + \Delta y$. The range of likely values of z will be rather smaller than from $(z - \Delta x - \Delta y)$ to $(z + \Delta x + \Delta y)$. We therefore combine the quantities in quadrature to reflect the probable range of values.

Multiplying or dividing quantities:

When two quantities are multiplied or divided, you need to combine the *relative* uncertainties.

The relative uncertainty in $z = (x \times y)$ [or $(x \div y)$] is given (combining the relative errors in quadrature) by:

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

Question 3

Write down the equation for the absolute uncertainty in transverse speed in terms of the relative uncertainty in distance and proper motion.

Your task now is to insert a formula for the uncertainty in transverse speed of the first star in the list (Sirius) in the relevant cell. (In the spreadsheet of Figure 3, this would be cell G10.)

The equation that you obtained in Question 3 is somewhat more complicated than those that you have used so far, so we'll guide you through the process of writing the corresponding spreadsheet formula. In the following we will assume that the spreadsheet has the row and column layout as shown in Figure 3 and give the appropriate cell references.

The uncertainty in transverse speed will be held in cell G10. The equation we want to use is (see Box 3 and the answer to Question 3)

$$\Delta \upsilon_{\text{trans}} = \upsilon_{\text{trans}} \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta \mu}{\mu}\right)^2} \qquad (1)$$

■ For the first star in the list (Sirius) what are the cell references of υ_{trans}, d, Δd and μ?

❑ The cell references are:

υ_{trans} is in cell F10, d is in cell C10, Δd is in cell D10, μ is in cell E10.

Thus we have cell references for all of the terms that appear on the right-hand side of Equation 1 with the exception of $\Delta\mu$. However a value of $\Delta\mu$ is given in the note on Table 1 – the proper motion has an uncertainty of ±1 milli-arcsec yr^{-1}.

So all of the variables that appear on the right-hand side of Equation 1 are known. The task now is to write a spreadsheet formula that represents the mathematical relationship between these variables. To do this you need to know the following spreadsheet conventions:

The square of a number is calculated using the operation '^2'. So, for instance, a cell that calculates the square of the quantity in cell B3 would be written as `=B3^2`.

The square root of a number is found by using the operation 'SQRT()': this returns the square root of the number inside the brackets. For example, a cell that calculates the square root of the quantity in cell F10 would be written as `=SQRT(F10)`.

Question 4

Write down the formula that calculates the uncertainty in transverse speed using the cell references given above.

Now that you have the formula for the uncertainty in the transverse speed insert it into the appropriate cell on your spreadsheet. As before, check the result is correct before replicating the cell for all stars.

You have now obtained results for distance and transverse speed in your spreadsheet. The data in the spreadsheet may, however, not have the correct number of significant figures. Although it is possible to adjust the number of decimal places displayed in any given cell on a spreadsheet, the correct number to use is not apparent until you have completed the uncertainty calculations.

Question 5

Rather than altering the formatting of each cell, which is very time consuming, finish off the activity by recording your final results in Table 3 below. Make sure that you quote your results to an appropriate number of significant figures.

Table 3 An empty table in which to record the results of the activity. For use with Question 5.

Star name	**Distance /pc**	**Uncertainty in distance/pc**	**Transverse speed /km s^{-1}**	**Uncertainty in transverse speed/km s^{-1}**
Alpha CMa (Sirius)				
Alpha Ori (Betelgeuse)				
Beta Ori (Rigel)				
Alpha Tau (Aldebaran)				
Alpha UMa (Dubhe)				
Kapteyn's star				

Here are a couple of more tricky questions that relate to the data you have just derived.

Question 6

It is possible to increase the accuracy of one of the measured quantities (parallax or proper motion) if they are made with a ground-based telescope, without using a different telescope. State which quantity this is correct for and explain why.

Question 7

Would it be more worthwhile making further observations to obtain a better result for the transverse speed of (a) Betelgeuse, or (b) Rigel? Explain your answer.

Notes

Note 1

The equation that needs to be used is that which calculates distance from the parallax in milli-arcsec. The relationship between parallax and distance is

$$d/\text{pc} = \frac{1}{p/\text{arcsec}}$$ (*An Introduction to the Sun and Stars*, Equation 3.7)

In this case the data are given in terms of milli-arcsec, so the appropriate equation is

$$d/\text{pc} = \frac{1000}{p/\text{milli-arcsec}}$$

If the spreadsheet is laid out as shown in Figure 3, the parallax (p) of the first star in the list (Sirius) is in cell B10, and the formula for distance needs to be inserted into cell C10. The formula in cell C10 should be `=1000/B10`.

Note 2

The equation that relates the uncertainty in distance d to the uncertainty in parallax p is

$$\Delta d = d\frac{\Delta p}{p}$$

If the spreadsheet is laid out as shown in Figure 3, the parallax (p) of the first star in the list (Sirius) is in cell B10, the distance is in cell C10 and the formula for the uncertainty in distance needs to be inserted in cell D10. Since the uncertainty in parallax Δp has a numerical value of 1 milli-arcsec, the formula that should be inserted in cell D10 is `=C10*(1/B10)`. (This could be simplified to `=C10/B10`, but that has the disadvantage that it is not as easy to see the relationship between the spreadsheet equation and the algebraic equation.)

Note 3

The equation that relates transverse speed to proper motion and distance is

$$v_{\text{trans}} = d\mu$$

For this speed to be expressed in km s^{-1}, d must be given in km and μ in rad s^{-1}. In the spreadsheet however, we have a distance expressed in parsecs and a proper motion expressed in milli-arcsec per year. So these value have to be multiplied by conversion factors of 3.09×10^{13} and 1.54×10^{-16} respectively if they are to be used to calculate the speed in km s^{-1}. (See Question 2.)

If the spreadsheet is laid out as shown in Figure 1, the distance (d) of the first star in the list (Sirius) is in cell C10, the proper motion is in cell E10 and the formula for the transverse speed needs to be inserted in cell F10.

The formula that should be written in cell F10 is

```
=(C10*3.09E+13)*(E10*1.54E-16)
```

You could simplify this expression to `=4.76E-03*C10*E10`, but as in the case of the distance calculation, this would make it more difficult to see the logic behind your calculation and is not recommended.

Answers to questions

Question 1

The distances of the nearer stars are known quite accurately (Sirius to better than 1%), but the method becomes less accurate for greater distances. This is because the fixed accuracy limit of 1 milli-arcsec is applied to smaller angles, leading to an uncertainty of over 20% for Rigel. The parallax method thus has an upper limit of a few hundred parsecs if determined to ±1 milli-arcsec.

Question 2

(a) To convert a distance expressed in parsecs into kilometres it is necessary to multiply by a factor of 3.09×10^{13} km pc^{-1} (see Box 2).

(b) To convert a proper motion expressed in milli-arcsec yr^{-1} into rad s^{-1} it is necessary to multiply by a factor of 1.54×10^{-16} radians s^{-1} (see Box 2).

Question 3

Start by looking at the equation $\upsilon_{\text{trans}} = d\mu$. The two quantities are multiplied together. The rule for multiplication and division is to combine the *relative* uncertainties.

In this case, the relative uncertainty in transverse speed is given by:

$$\frac{\Delta\upsilon_{\text{trans}}}{\upsilon_{\text{trans}}} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2}$$

so the absolute uncertainty in transverse speed is

$$\Delta\upsilon_{\text{trans}} = \upsilon_{\text{trans}}\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2}$$

Question 4

The equation for uncertainty in transverse speed is

$$\Delta\upsilon_{\text{trans}} = \upsilon_{\text{trans}}\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\mu}{\mu}\right)^2} \tag{1}$$

and the cell references or values of the variables on the right-hand side of this equation as shown in Table 4:

Table 4 The cell references or numerical values of variables from Equation 1.

Variable	υ_{trans}	Δd	d	$\Delta\mu$	μ
cell reference	F10	D10	C10	-	E10
numerical value	-	-	-	1	-

Thus the formula used to express the right hand side of Equation 1 is

```
=F10*SQRT((D10/C10)^2+(1/E10)^2)
```

Note that it is essential that the numbers of left- and right-hand brackets are equal.

Question 5

The final results, quoted to an appropriate number of significant figures are shown in Table 5.

Table 5 Summary of results (completed version of Table 3).

Star name	Distance /pc	Uncertainty in distance/pc	Transverse speed /km s^{-1}	Uncertainty in transverse speed/km s^{-1}
Alpha Cma (Sirius)	2.64	0.01	16.81	0.05
Alpha Ori (Betelgeuse)	132	17	18.4	2.5
Beta Ori (Rigel)	238	57	2.3	1.3
Alpha Tau (Aldebaran)	20.0	0.4	19.0	0.4
Alpha UMa (Dubhe)	37.9	1.4	25.4	1.0
Kapteyn's star	3.92	0.02	161.6	0.6

Question 6

The accuracy of the measurement of a position depends on the size of the telescope and the size of the 'blur' due to atmospheric seeing, so it is not possible to improve these using the same telescope. The apparent position of a star changes due to parallax, with the star tracking out a small ellipse on the sky over the course of a year. Observations need to be made approximately six months apart to observe the maximum apparent change in position. The component of the true motion of the star through space causes it to move across the sky with a given proper motion. If the time interval between observations is extended, this change in position will be larger and the proper motion can be determined more accurately.

Question 7

More observations could significantly improve the accuracy of the proper motion of Rigel but not the parallax (see answer to Question 6). The uncertainty in the transverse speed results from a combination of the relative uncertainties in the distance (i.e. parallax) and proper motion. Table 6 shows the values for Betelgeuse and Rigel.

Table 6 Relative uncertainties in distance and proper motion for Betelgeuse and Rigel.

	Betelgeuse	Rigel
relative uncertainty in distance, $\Delta d/d$	13%	3.4%
relative uncertainty in proper motion, $\Delta\mu/\mu$	24%	50%

For Betelgeuse, the larger relative uncertainty is in the parallax so improving the proper motion accuracy would not make a significant difference to the accuracy of the transverse speed. However, for Rigel the relative uncertainty in the proper motion is larger than for the parallax so the uncertainty in the transverse speed could be significantly reduced if observations were made for longer (unfortunately this cannot be done with Hipparcos as the satellite is no longer operational!).

Resources

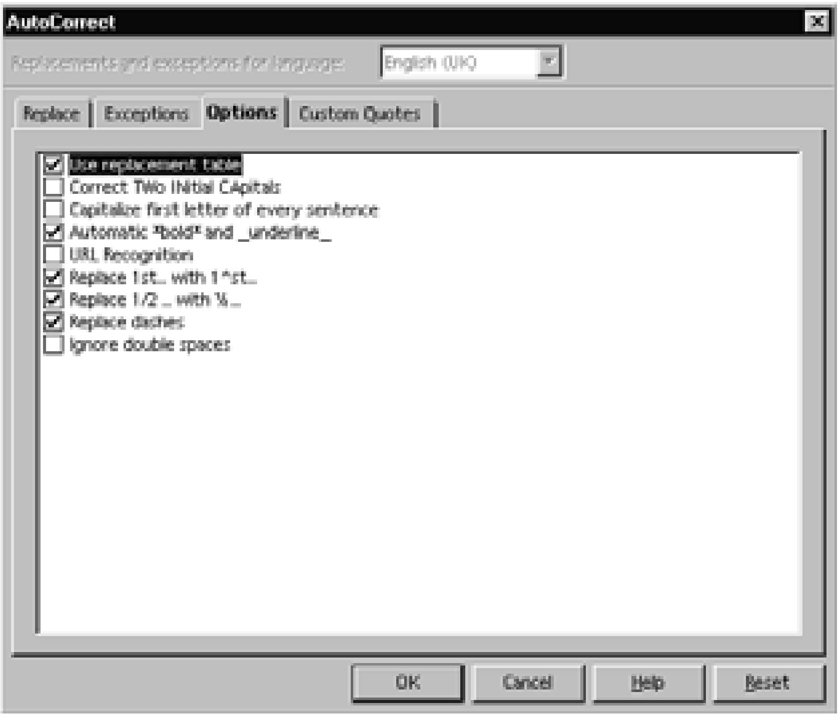
ESA Hipparcos catalogue http://www.rssd.esa.int/Hipparcos/

Appendix A Spreadsheet technique: autocorrect

Most commercial software for word processing and spreadsheets includes automatic checks for 'mistakes', such as ensuring that sentences start with a capital letter, or preventing more than one capital letter in a word. In addition, the software may recognize website or e-mail addresses and produce automatic links.

While these functions can be very useful, they may interfere with the way you want a document to look. The layout shown in Figure 3 is not possible without turning off some of these options (e.g. 'Alpha CMa' is automatically corrected to 'Alpha Cma' and 'km s^-1' is converted to 'Km s^-1'). It is possible to switch off some of the automatic corrections in the StarOffice spreadsheet in the following way:

Select Autocorrect from the Tools menu and you will see the following box.



You can click on the small squares to toggle on or off any of the autocorrect functions. The screen shown here will allow you to reproduce exactly the spreadsheet layout shown in Figure 3.

Appendix B Spreadsheet technique: named constants

You have probably found out by now that formulae in spreadsheets can be quite tricky to write. Anything that can be done to clarify and make your formulae easier to understand will be useful to both yourself and to anyone reading or using the spreadsheet.

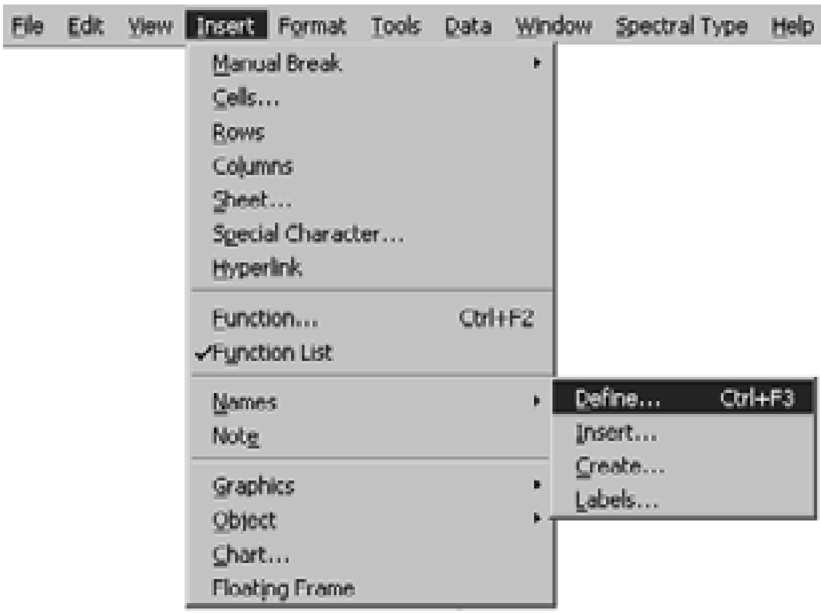
One way to improve the readability of your sheet is to use named constants instead of writing numbers or obscure cell references in your formulae.

Here, an area has been created to contain values for these conversion constants. Notice that each constant has been labelled in column A, and the units are noted in column B.

Now, you can give each cell containing a constant a name.

Highlight the cell containing the value that you want to give a name (cell B5 for the parsec to km conversion in this example).

From the StarOffice menu select Insert | Names | Define...

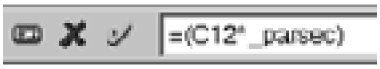


In the resulting dialog box, enter the name `_parsec` and press OK.

(You don't have to start the name with an underscore, but it avoids confusion with cell references and makes your formulae easier to follow. Other people using the spreadsheet will know that this is a named constant.)

You can now refer to this cell using either the reference $B$5 or the name _parsec.

So, for example, to convert a value (in parsecs) contained in cell C12 into km, you could enter the formula as follows:



S282 Astronomy

Wien's law

Study time: 1 hour

Summary

In this spreadsheet activity you will investigate spectra from a number of different types of star.

You will use Wien's displacement law to obtain an estimate of the surface temperatures of the stars and compare this with the temperature derived by fitting the spectrum with the Planck (black-body) function. This activity is related to work in Chapter 3 of *An Introduction to the Sun and Stars*. In particular, you should have read Sections 1.3.2 and 3.3.2 of this book before starting this activity.

Learning outcomes

- Understand Wien's displacement law.
- Appreciate the differences in stellar spectral continuum shapes.
- Handle uncertainties.
- Develop your spreadsheet skills.

Background to the activity

In this activity you will use an existing spreadsheet to explore how well stellar spectra can be modelled as black-body spectra.

Wien's displacement law

Most normal (i.e. main sequence) stars have spectra that approximate to that of a black-body source.

As described in Section 1.3.2 of *An Introduction to the Sun and Stars*, Wien's displacement law relates the temperature of a black-body source to the peak wavelength in the spectrum in a very simple way:

$$\lambda_{\text{peak}}/\text{m} = \frac{2.90 \times 10^{-3}}{T/\text{K}}$$

So an estimate of a given star's temperature can be made by measuring its spectrum and finding the peak wavelength.

The black-body spectrum: Planck's radiation law

Wien's law gives a simple relationship between temperature and peak wavelength of the spectrum. For a complete understanding of the shape of a black-body spectrum it is necessary to turn to quantum mechanics. Again, there is a law (called Planck's radiation law) that gives the shape of the spectrum for a black-body source at a given temperature. (If you are interested in the mathematical

details of this law see Appendix A – although do note that you are *not* expected to understand the mathematical details of this law.) Although *you do not need to learn this law* you will use it as part of the prepared spreadsheet to test how well a stellar spectrum compares with a true black-body spectrum. Planck's law allows us to calculate the flux density emitted from the surface of a black body.

The shape of the black-body spectrum (also known as the Planck curve) depends only on temperature and can therefore be fitted to the actual spectral data to give a more precise measurement of temperature. Note that the temperature derived in this way is based on the shape of a large part of the spectrum rather than just the peak wavelength, and since the star's spectrum only approximates a black body, the two results may be different.

The activity

The spreadsheet supplied contains data for a selection of stars of different spectral types. The spectra used in this activity have been selected from a catalogue of stellar spectra obtainable from *The Electronic Universe* at
http://zebu.uoregon.edu/spectra.html

They were taken using a telescope and spectrograph at an Earth-based observatory and the spectra shown have been corrected for the variable transmission of the Earth's atmosphere. This is achieved by observing a star with a known spectrum (a 'standard star') under identical conditions, which will therefore be affected in the same way by the atmosphere. If the standard star has an observed spectrum with spectral flux density $S_{\mathrm{obs}}(\lambda)$ and a true spectrum (unaffected by the atmosphere) $S_{\mathrm{true}}(\lambda)$, then the true spectrum of any star $F_{\mathrm{true}}(\lambda)$ can be determined from its observed spectrum $F_{\mathrm{obs}}(\lambda)$ by:

$$F_{\mathrm{true}}(\lambda) = F_{\mathrm{obs}}(\lambda) \times \frac{S_{\mathrm{true}}(\lambda)}{S_{\mathrm{obs}}(\lambda)}$$

If you haven't already done so, open the spreadsheet 'Wiens_Law.sxc' from the course DVD in the usual manner.

- Start the S282 Multimedia guide program and open the 'Stars' folder, then click on the icon for this activity ('Wien's law').
- Press the Start button to access the folder on the DVD containing the StarOffice and Excel versions of the raw data file.
- Open the file you wish to use by double-clicking on it.

The spreadsheet

The spreadsheet is divided into a number of sections; below the headings are three yellow areas used for calculations.

The first of these, headed Constants, contains the values of a number of constants that will be used in the calculations.

Constants		
Speed of light	3.00E+08	m/s
Boltzmann	1.38E-23	J/K
Planck	6.63E-34	Js
nm	1.00E-09	
Wien constant	2.90E-03	Km

Although good for calculations spreadsheets tend not to be good at keeping track of units, so the units of each constant have been included explicitly in the third column. Also, note that in the spreadsheet the units have been expressed in the form ‘m/s’ rather than the preferred convention ‘m s^{-1}’, and there should also be spaces between ‘Js’ and ‘Km’ to avoid any ambiguity. (‘K m’ means ‘kelvin metres’ – do not mistake this for ‘km’, kilometres.)

The second area, Wien’s Law, contains two white cells into which you will enter your estimates of peak wavelength and its associated uncertainty, and two yellow cells that will give the results of the Wien’s law temperature calculation again with units and associated uncertainty:

Wien's Law				
Peak Wavelength		+/-		nm
Temperature		+/-		K

The actual spectral data are listed lower down, in the columns labelled Wavelength and Intensity:

Spectral Type O5				
Wavelength (nm)	Intensity		Planck fit	Scaled Planck fit
351.0	521.2			
351.5	526.93			
352.0	528.07			

To the right of those columns are some further columns that will contain the results of the Planck curve fitting calculations.

You can select individual spectra for investigation using the tabs at the lower left-hand corner of the spreadsheet:

42	364.5	466.67
43	365.0	465.93

O5 / B6 / A2 / F0 / F7 / G2 / K0 / K5 / M2 / Spectra

Each sheet on the spreadsheet contains the data for a different spectral type, from O5 to M2. You can select a sheet simply by clicking on the tab. Here, the tab indicates the sheet containing the spectrum for a star of spectral type O5.

Estimate the peak wavelength

From the spectral type tabs select the sheet containing the K5 spectrum. Each sheet has the same layout with the spectral data in the Intensity column and a graph of the spectrum plotted to the right of the data columns.

By looking at the graph make an estimate of the peak wavelength. As the data are noisy (rather than a smooth curve) this will not necessarily be simply the highest data point: rather, you should try to estimate the overall maximum of the distribution. As this is not an exact process you should also make an estimate of the *uncertainty* of your value – how accurately can you judge the peak wavelength?

Record these values in column 2 of Table 1 (p. 5), and then enter them into the Wien’s Law area of the spreadsheet. (Enter the wavelength, in nm, into cell G5 and the uncertainty, again in nm, into cell I5.)

Calculate the temperature

Once the wavelength has been entered the temperature is calculated using the following rearrangement of the Wien's law equation:

$$T/\mathrm{K} = \frac{2.90 \times 10^{-3}}{\lambda_{\mathrm{peak}}/\mathrm{m}}$$

The result appears in cell G8. Record this temperature and its uncertainty in column 3 of Table 1.

■ How would you convert the uncertainty in wavelength into an uncertainty in temperature?

❑ Looking at the formula the temperature is calculated as a constant divided by the wavelength. The relative uncertainty in the result will be the same as the relative uncertainty in the wavelength. Thus, if the wavelength is known to an accuracy of 10% (i.e. a relative uncertainty of 0.1) the calculated temperature will also be accurate to 10%. (Handling uncertainties was addressed in more detail in the activity 'Stellar distance and motion'.)

The sheet calculates the uncertainty in temperature in this way and places the result in cell I8.

Use the Planck radiation law to obtain best fit to a black-body spectrum

Once the estimated temperature has been calculated you can use Planck's radiation law to calculate a theoretical spectrum, which can be compared to the actual spectrum.

In the area of the spreadsheet Planck Fit enter the temperature you obtained using Wien's law. (Note that the units of 'Nm' in the spreadsheet should be 'nm'. This error shows what happens with the spreadsheet's autocorrect function – see Appendix A of the activity 'Stellar distance and motion' to learn how to switch it off.)

The spreadsheet will draw the corresponding Planck curve in blue on the same graph as the stellar spectrum. You can now adjust this temperature until you obtain what you believe to be the best overall match to the shape of the spectrum. You will not be able to get an *exact* match to the data, but you should try to get the best overall agreement you can. Note that the black-body spectrum is only scaled to place it on the same graph; it could be raised or lowered to obtain a better fit. You will need to use your judgement to estimate which temperature produces the best fit. Enter this value in column 4 of Table 1 with the corresponding peak wavelength in column 5.

Also, you can estimate the uncertainty in these values by estimating the range of temperatures over which you are able to obtain an acceptable fit. Use these values to define the uncertainty in temperature and the corresponding uncertainty in the peak wavelength, and enter the data in columns 4 and 5 of Table 1.

Write down any comments you have on your ability to perform the measurements and the agreement (or discrepancy!) between the results.

Repeat these activities for the other spectral types and complete the remaining rows in Table 1. For those at either end (O5 and M2) the peak appears to be outside of the measured wavelength range so you will only be able to make an estimate of the upper or lower limit to the temperature. However, the best fit using the Planck function may give you a better estimate. (A completed Table 1 is given 'Answers to questions' section found at the end of this activity.)

Table 1 An empty table to record your results.

1	2	3	4	5
Spectral type	**Peak wavelength measured/nm**	**Surface temperature derived from Wien's law/K**	**Surface temperature derived from fitting Planck curve/K**	**Peak wavelength obtained from fitting Planck curve/nm**
O5				
B6				
A2				
F0				
F7				
G2				
K0				
K5				
M2				

Comments

O5	
B6	
A2	
F0	
F7	
G2	
K0	
K5	
M2	

Question 1

The results you have obtained for the best fit temperature using the Planck function are unlikely to be the same as those using Wien's law and it is even possible that your estimated uncertainties may not overlap. Can you give any reason for this?

Question 2

Compare your results in Table 1 with Table 3.2 in *An Introduction to the Sun and Stars*. How well do your values for the temperatures of the different spectral types correspond with those given in the table?

Question 3

The data for the spectra in the spreadsheet contain a number of gaps, particularly towards the shorter wavelength end. What is the cause of these gaps?

Question 4

Estimating the temperature of the O and M stars is made difficult because the peaks of their spectra lie outside the visible range. Can you give two possible reasons why the spectra do not extend further into the ultraviolet or infrared?

Endnote

In this activity you have attempted to determine stellar temperatures by applying Wien's law and Planck's law to real stellar spectra. You will have recognized that although the shape of stellar spectra approximate to black-body curves there are significant differences at some wavelengths, which can lead to erroneous estimates of temperature. In particular, you will have recognized that your estimates of uncertainties based on the accuracy of your measurement of a quantity do not always provide a reliable estimate of the true uncertainty. This is because you are making an implicit assumption when using those measurements (i.e. that the stellar spectrum is a black body) which is not true. Temperatures derived by studying the relative strengths of spectral lines will often be more reliable (see Section 3.3.2 in *An Introduction to the Sun and Stars*).

Answers to questions

Question 1

You would only expect to get the same answer if the star's spectrum was *the same* as a black-body and there were no observational errors.

Planck function fitting is the best technique since the whole of the spectrum is used rather than just one part, i.e. the peak.

Comment

For the B6, A2 and F0 stars the difficulty with fitting the Planck function arose because the spectrum appears to be too low (compared with the Planck function) at short wavelengths (i.e. less than ~400 nm). In this part of the spectrum the hydrogen Balmer lines (see Section 3.3.2 of *An Introduction to the Sun and Stars*) bunch closer and closer together, corresponding to transitions of electrons from energy level $n = 2$ to higher and higher levels. These transitions result from absorption of radiation by the stars' atmospheres (see Section 1.3.2 in *An Introduction to the Sun and Stars*). At wavelengths shorter than 365 nm photons can ionize the hydrogen and so are relatively easily absorbed, causing a dip in the spectrum called the Balmer discontinuity. Other significant deviations from

black-body spectra occur when molecules are present, such as TiO where the combination of many lines forms bands in the spectra of type M stars.

Question 2

Table 2 shows and example of the results that you might obtain from performing this activity. You may find that your answers are in some cases rather different from those in Table 3.2 in *An Introduction to the Sun and Stars* even allowing for the uncertainties in your results. This is likely to be due to the difference between the true spectral shapes of the stars and the black-body curves used for comparison (see comment above). This absorption can also affect the position of the peak of the spectrum giving an erroneous solution from Wien's law (e.g. for the B6 star).

Table 2 Example of the results obtained by performing this activity.

1	2	3	4	5	
Spectral type	**Peak wavelength measured/nm**	**Surface temperature derived from Wien's law/K**	**Surface temperature derived from fitting Planck curve/K**	**Peak wavelength obtained from fitting Planck curve/nm**	**Temperature derived from Table 3.2 in *The Sun and Stars*/K**
O5	< 350	> 8300	$45\,000 \pm 5\,000$	64 ± 6	40 000
B6	380 ± 10	7600 ± 200	$14\,000 \pm 1\,000$	205 ± 20	15 000
A2	400 ± 10	7250 ± 200	9500 ± 500	305 ± 20	9500
F0	405 ± 10	7200 ± 200	7500 ± 500	390 ± 30	7400
F7	440 ± 20	6600 ± 300	6600 ± 300	440 ± 20	6500
G2	450 ± 10	6450 ± 150	6100 ± 300	475 ± 25	5800
K0	480 ± 20	6050 ± 250	5400 ± 200	540 ± 20	4900
K5	610 ± 20	4750 ± 150	4400 ± 200	660 ± 30	4100
M2	> 800	< 3600	3200 ± 300	910 ± 90	3200

Comments

O5	Upper limit to peak wavelength gives lower limit to temperature
B6	Large discrepancy between T derived from peak wavelength and Planck curve fit. Hard to fit Planck curve – large amount of absorption at short wavelength end of spectrum
A2	T from peak wavelength lower than from Planck fit. Absorption at short wavelength end of spectrum prevents fitting Planck curve well
F0	Impossible to fit Planck curve well so large error results
F7	Hard to fit Planck curve
G2	None
K0	Large discrepancy between T obtained from Wien's law and Planck fit
K5	Good Planck function fit obtained
M2	Hard to tell if peak has been reached within observed spectrum. Lower limit to peak wavelength gives upper limit to temperature

Question 3

The observations were made with a ground-based telescope and spectrometer. The atmosphere absorbs radiation at certain wavelengths (see Figure 1.38 in *An Introduction to the Sun and Stars*) and the gaps probably occur where the atmospheric transmission is not sufficiently high to provide good data.

Question 4

(i) The detector may not be sensitive to radiation at these wavelengths. (This is likely to be the case for the infrared end of the spectrum.)

(ii) The atmosphere may absorb the radiation. (This is likely to be the case at the ultraviolet end of the spectrum.)

Resources

The Electronic Universe http://zebu.uoregon.edu/spectra.html

Appendix A The black-body spectrum

Planck's law describes the shape of a black-body spectrum and allows us to calculate the spectral flux density emitted by a black-body surface using:

$$F_\lambda/(\mathrm{W\,m^{-2}\,nm^{-1}}) = \frac{2\pi hc^2}{\lambda^5}(\mathrm{e}^{hc/\lambda kT} - 1)^{-1}\,\Delta\lambda \quad \text{where } \Delta\lambda \text{ is } 10^{-9}\text{ m}$$

In this equation, λ is the wavelength, h is the Planck constant (6.63×10^{-34} J s), c is the speed of light (3.00×10^{8} m s^{-1}), k is the Boltzmann constant (1.38×10^{-23} J K^{-1}) and T is the temperature. e is a number (2.718…), which is raised to the power $hc/\lambda kT$ in this equation.

S282 Astronomy

Spectral classification of stars

Study time: 3 hours

Summary

In this activity you will learn how to classify main sequence stars according to their spectra. The activity is based around a software package that allows you to view and compare stellar spectra. You will measure the wavelengths and intensities of absorption lines and study how the prominence of certain lines varies across spectral classes.

The software package also includes a simulation of an astronomical telescope and you will use this to obtain spectra of some of the stars in the Pleiades. The analysis of these data will allow you to measure the distance to this star cluster.

Before doing this activity you should have read as far as the end of Chapter 3 of *An Introduction to the Sun and Stars*.

The activity is based on a software package called the Virtual Educational Observatory (VIREO) developed by a group called Contemporary Laboratory Experiences in Astronomy (CLEA) that is based at Gettysburg College, Pennsylvania, USA.

Learning outcomes

- With reference to a spectral atlas, the ability to classify main sequence stars according to the Harvard Spectral Classification scheme.
- Better understanding of the technique of spectroscopic parallax, including the ability to determine the distance to a star using this method.
- An appreciation of the factors that are important in making efficient use of telescope time.

Background to the activity

The Harvard Spectral Classification scheme (O, B, A, F, G, K, M) (*An Introduction to the Sun and Stars*, Section 3.3.2) is based on correlating absorption line strengths with photospheric temperature. Since absorption lines might be considered to be fine detail in the spectrum, you will appreciate that you will need to pay quite close attention to the spectra.

The astronomers who developed the Harvard system, Annie Jump Cannon and E.C. Pickering, classified an astonishing 225 300 stars in their original 'Henry Draper Catalogue'. This was published from 1918 to 1924. You will be studying some 'unknown' spectra that relate to stars in this original catalogue, which are identified by the letters 'HD'.

In this activity we will consider only the spectra of main sequence stars. It is important to note, however, that the software package contains spectra of stars that are not on the main sequence, and that the classification of spectra is given

by an extended scheme called the MK system. The MK system describes spectral classes in the same way as the Harvard scheme, but includes an indication of the stars' luminosity. The luminosity is indicated by a roman numeral (I, II, III, IV or V) called the luminosity class, with I as the highest and V as the lowest (see *An Introduction to the Sun and Stars*, Section 4.4.2). Main sequence stars are all of luminosity class V, and in this activity you will only need to consider spectra that have this luminosity class (e.g. G0 V, K5 V, M5 V, etc.).

Another convention that you need to be aware of when carrying out this activity is that all wavelengths are measured in units of 10^{-10} m. This unit is called the ångström, and has the symbol Å. The ångström is commonly used in astronomy, although you may not have come across it before, since in this course we express wavelengths in the visible part of the spectrum in units of nanometres (nm).

■ How many ångströms are there in a nanometre?

❑ 10. Since a nanometre is 10^{-9} m, and one ångström is 10^{-10} m, there are $10^{-9}/10^{-10} = 10$ ångströms in one nanometre.

This activity requires several windows to be open at once, and is easier to use if you have your computer set to a high screen resolution. If you know how to change the screen resolution of your computer you may want to do that before starting the activity, but it is not essential that you do so.

This is a fairly long activity so it has been divided into parts. The estimated study times are given for the individual parts.

Setting up the VIREO software package

- If you have not already installed the VIREO package, start the S282 Multimedia guide and then click on The spectral classification of stars in the 'Stars' folder. Follow the instructions to install VIREO. This may take several minutes to load.

VIREO includes a range of default options which include a simulation of the process of applying for telescope time to access large telescopes before observing. Time is not always "awarded" so it is useful to turn off this this option the first time you use VIREO. To do this:

- start VIREO using the shortcut on your desktop. The opening screen of the CLEA package will be displayed.
- Click on its menu item File | Log In...A new window called Student Accounting will open.
- Type Instructor (not case sensitive) in the box labeled Student #1 and click OK. Answer YES to the question Have you finished logging in? A small dialog box will appear, asking for a password. The default password is CLEA (case sensitive). Enter it in the box and press OK. A window called Optical Simulation Options will open with the General tab selected.
- Click on the box labelled Restrict Scope Access to remove the tick. Click OK and click YES in the dialog window that follow.
- Exit VIREO using the File | Exit Observatory menu item.

The spectral classification of main sequence stars

Part 1 Familarization with the display of stellar spectra

This part of the activity should take about 15 minutes.

- Start VIREO using the shortcut on your desktop.
- The opening screen of the CLEA package will be displayed. Click on its menu item File | Log In...
- A new window called Student Accounting will open. You should enter your in the box labeled Student #1. The software uses your name as a way of naming folders for containing the results that you obtain. The software was originally written for teaching in undergraduate teaching laboratories – hence the option to login as a group of students at a particular table. You only need to fill in your name, press OK and then confirm that you have finished logging in.

You will now see the main title screen for the Virtual Educational Observatory (VIREO). It is from this screen that you can choose a number of different activities. Here we will be using the Classification of Stellar Spectra exercise.

In this part of the activity we will be interested in using the spectral classification tool.

- Select the menu item File | Run exercise | "Classification of Stellar Spectra". This opens a new window entitled The Classification of Stellar Spectra. Select the menu item Tools | Spectral Classification and take a moment to look at the layout of the new screen that is displayed (titled Classify Spectra). You should see that the horizontal axis has a scale from 3700 to 4700 ångströms – this is the wavelength scale for displayed spectra. Change this range to 3900 to 4500 ångströms for this activity by selecting the menu item File | Display | Spectral range...and set the minimum and maximum wavelengths to 3900 and 4500 ångströms using the slider, then click OK .This wavelength range is typical for stellar classification.

- What part of the spectrum does this wavelength range correspond to? Look back to *An Introduction to the Sun and Stars* Figure 1.36 and note the corresponding colour(s).

❑ The wavelength range extends across the violet part of the spectrum.

The vertical axis shows the spectral flux density from the star in arbitrary units. This axis is labelled 'Normalized Intensity'. Note that the axis is without any numerical values – we will return to consider this in more detail when we display the first spectrum.

You should also note that the screen is split into three panels. Each of the three panels can be used to display a spectrum. The central panel will hold the unknown spectrum that we wish to classify, and the panels above and below it can be used to display known reference spectra. This layout, with an unknown spectrum between two known spectra allows comparisons to be made between spectra, and greatly helps in the classification process.

The software package contains 25 'unknown' stellar spectra. Later in this activity you will classify all of these spectra in turn, but to start with, we will describe in detail how to go about classifying the first two stars from this list.

- To load the first spectrum for study select the menu item File | Unknown Spectra | Program List. This opens another window with a list of stars.
- Select the first star in the list – HD 124320 – and double-click to open the spectrum. Notice that the spectrum of HD 124320 now appears in the central panel of the classification window.
- The spectrum is shown as a graph of 'Normalized Intensity' against wavelength. In this software package, normalized intensity means that the highest intensity is given the numerical value of 1.00 and that all other intensities are scaled accordingly.
- Click on the highest point of the curve to confirm this. The intensity value will be shown at the bottom of the window. Note also that when you click on the spectrum to make a measurement, the wavelength is also displayed at the bottom of the screen. Click on the horizontal axis to check that it represents zero intensity.

The spectrum appears as a 'plateau' of slowly varying high intensity into which cut four or five deep notches. The notches are absorption lines and the 'plateau' part is the continuum. In our study of spectral classification we will be interested in both of these components.

- Note that you can also view the spectrum in a 'photographic' form by using the menu: File | Display | Grayscale "Photo". This reveals the absorption lines as narrow dark bands.
- Switch back to the graphical display File | Display | Intensity Trace.
- Click on the lowest point of each of the three deepest absorption lines. Record their wavelengths and normalized intensities. Your results should be close to wavelengths 3970, 4102, 4342 Å and normalized intensities 0.27, 0.29, and 0.27 respectively.

Question 1

For the spectrum that is currently displayed:

(a) Determine the normalized intensities at 4050 and 4400 Å.

(b) Roughly estimate the wavelength at which the continuum seems to have its maximum value.

(c) Decide if the star will appear red or blue.

(d) Surmise if this star has a hot or cool photosphere (say, in comparison to that of the Sun).

Although we have examined this spectrum and concluded that the star probably has a higher photospheric temperature than that of the Sun, we cannot assign a spectral class to the star unless we compare it to some known 'standard' spectra. In the next part of the activity you will classify this spectrum and those of the other unknown stars that are given in the Program List.

Part 2 The classification of stellar spectra

This part of the activity should take about 45 minutes.

This part of the activity is a study of the distinguishing characteristics of the spectra of main sequence stars. You will compare some unclassified stellar spectra with reference spectra to enable you to determine the correct spectral classifications. You will start by attempting to classify the spectrum of HD 124320, so ensure that you have this spectrum displayed (look back to Part 1 to see how to do this if you have taken a break from the activity).

- Click on the menu item File | Atlas of Standard Spectra.
- In the new window select Main Sequence by double-clicking.

Reference spectra from the main sequence catalogue will now be shown in the panel labelled "Standards" on the right-hand side of the window. The spectral classifications shown include the Roman numeral 'V', indicating the luminosity class in the MK classification scheme. Since this activity is based only on main sequence stars you may omit the 'V' when writing down any classifications that you arrive at.

- You can scroll through this catalogue using the scroll bar. Astronomers often refer to spectral classes as 'early' or 'late'. The early classes (O, B, A) are at the top of this catalogue and the late classes (K, M) at the bottom. Click on a reference spectrum to display it in the top panel, above the unknown spectrum. The next spectrum in the atlas will be displayed in the bottom panel. Notice how the overall continuum shape changes through the spectral sequence.

In light of what you've observed so far consider the following points.

■ What does the varying shape of the continuum indicate?

❑ It is caused by the systematic change in photospheric temperature across the spectral classes.

■ Use the location of the peak in the continuum to decide which star has the coolest photosphere and which has the hottest.

❑ The coolest photosphere is from class M5 and the hottest O5.

Although this is slightly ambiguous (the nearby spectra are very similar) you will have seen that the continuum peak for O5 is off the short wavelength end of the spectrum (in the ultraviolet) and M5 has its peak beyond the long wavelength end.

■ Scroll through all the spectra in the Classification Atlas again, and compare the selection of spectra available with Table 3.2 of *An Introduction to the Sun and Stars*. Note any differences.

❑ The two sets are very similar and the only difference is in the range of the sequence. Both start with O5, the first in the spectral sequence. However, this Atlas only runs as far as M5 whilst Table 3.2 extends to M8.

As you scanned through the Atlas you probably noticed that the strength of spectral lines varies through the sequence. If not, look once again at the Atlas, before considering the following question.

Question 2

Why does the strength of a spectral line change with spectral type? (To answer this question fully, you may need to refer back to *An Introduction to the Sun and Stars*, Section 3.3.2.)

Return to the O5 spectrum and note the narrow dip about one-quarter of the way from the long wavelength end. This absorption line grows stronger in the later spectral classes shown lower down the window.

■ Look at each spectrum in the atlas and decide in which spectral class this line is strongest.

❑ This line is strongest in the A1 class.

■ Click on the lowest point of the same absorption line (one-quarter of the way from the long wavelength end) on the three displayed spectra and note the wavelengths in each case.

❑ You should obtain values of approximately 4340 Å in all three cases.

So it appears as though this is the same spectral line. At this point it is useful to identify the line, i.e. to determine what element or ion is responsible for absorption at this wavelength.

This spectral classification software includes a table of the most prominent spectral lines, and we will now use this table to identify the line at 4340 Å.

- Click on the menu item File | Spectral Line Table. Place the cursor on the coloured title bar of the new window, hold down the left mouse button and drag the table off the classification window so that both windows are visible.
- Either scroll down the Spectral Lines window to find the closest match to this line, or simply click on the lowest point of the line, in the classification window, to get the software to find the match.

With this in mind consider the following queries.

■ Which line is the closest match to the feature at 4340 Å? What element does it arise from and what is the ionization state of this element?

❑ The line is described as HI (H gamma). This is the Hγ line of hydrogen, i.e. the third line in the Balmer series (see *An Introduction to the Sun and Stars*, Section 3.3.2). The HI indicates it arises from un-ionized (or 'neutral') hydrogen.

You are now ready to attempt a classification for HD 124320. The general principle is to find the reference spectra that provide the closest match to the unknown spectrum. Often the unknown spectrum will have an appearance that lies between those of two reference spectra. If it appears 'mid-way' between two reference spectra then you can assign it to the spectral class that lies mid-way between the classes of these reference spectra. If it is closer to one reference than the other then you should assign it to a class that it closer to one reference class than the other.

Of course, there is a degree of subjectivity to such classification, but with practice you should be able to classify a main sequence stellar spectrum to within two-tenths of a spectral class.

To proceed with the classification click on each of the standard spectra in turn and compare with the spectrum of HD 124320.

■ Which spectral types are the closest match to HD 124320?

❑ A1, A5 are both very similar to HD 124320. F0 is reasonably similar, but probably not as good a match as A1 or A5.

So, you would justifiably conclude that the classification of HD 124320 lies between A1 and A5. To investigate in more detail we can use another feature of

this software package – one which shows the difference between an unknown spectrum and a reference.

- Select the menu option File | Display | Show Difference. This subtracts the spectrum of HD 124320 point by point from the reference spectrum at the top of the window and displays the result at the bottom.

Question 3

Using the Show Difference function, now decide which spectral types are the closest match to HD 124320. If you had to attribute a spectral class (that might not be shown in the Atlas) to HD 124320, what would it be?

The difference plot is very close to zero except for one feature.

Question 4

Use the cursor to measure the wavelength and size of the discrepant feature identified in Question 3. Now use the menu item File | Spectral Line Table to identify the spectral line, and explain what the table entry means. Note that by double-clicking the mouse button on a table entry, you can obtain additional information about the line. What does the feature in the difference plot represent? Give as full a description of this line as you can.

- Record your classification of the spectrum of HD 124320 by clicking on the menu item Classification Results. A new window titled 'Observational Data' will open. Enter the class in the box labelled Sp. Type and the reasons for your choice in the box labelled Remarks and click OK. A message will appear to tell you the data have been added to the results list.

Note that all your results can be stored in this way – and this will be useful when you come to compare your spectral classifications with those given at the end of these notes. If you want to amend or print your table of results, use the menu item Tools | Results Editors | Observational Results | Display/Print/Save Text... which can be found on the main exercise window (you may need to move the Classify Spectra window to one side to access the main window). A new window entitled "Recorded Results" will appear. To print the results select List | Print. To amend an entry, double-click on it and select Edit from the pop-up menu.

Note that if you want to take a break from the activity, you must save the results in a file using the menu item Tools | Results Editors | Observational Results | Save Data... from the main exercise window. To reload data after a break go to Tools | Results Editors |Observational Results | Load Saved Data...

- Now inspect the CaII K line in more detail (see the answer to Question 4). Scroll through the reference spectra and note the behaviour of this line throughout the spectral sequence.

Question 5

Describe qualitatively how the CaII K line varies across the whole spectral sequence. Compare your findings against Figure 3.23 of Chapter 3 in *An Introduction to the Sun and Stars* (note that the term 'ionized calcium' in Figure 3.23 refers to the CaII K line).

This completes our work on HD 124320. Next you will use the software to classify the second unknown spectrum in the list, that of the star HD 37767.

- Go to the menu item File | Unknown Spectra | Next on List.

Question 6

(a) Using a similar approach as you adopted for HD 124320, estimate the spectral class of HD 37767.

(b) For the spectrum of HD 37767 list the wavelengths of all the spectral lines that you can find. Identify as many of these lines as you can.

You might find it useful switch to the grayscale or 'photo' display (File | Preferences | Display) to see the visual appearance of this line, and how the neighbouring reference spectra compare. You may also have difficulty identifying some lines because they overlap one another. You can use the Zoom feature to take a closer look, but you may not be able to unambiguously identify every line.

The final task in this part of the activity is to classify the remaining stars in the unidentified stars list. Although there are over 20 stars in the list you will find that the process of assigning stars to classes becomes much faster once you have had some practice. You should start with the stars HD 6111 and HD 5351 since the notes at the end of this activity describe in detail how the classes for these two stars are arrived at. Remember to record your results using the menu item Classification Results and, if you want to take a break, to save a file of your results using the menu item Tools | Results Editors | Observational Results | Save Data... from the main exercise window.

- Classify all of the unknown stars in the 'Unknown spectrum' list. You can obtain the list of stars by the menu item File | Unknown Spectrum | Program List.
- To select a particular star scroll down the pop-up window and double-click the star name. Then classify the star using the techniques that you have learnt in this activity. Make sure to record your results, and take careful note of any peculiarities that you come across as you work through the list.
- When you have finished your classification, you might like to print out your table of results using menu item Tools | Results Editors | Observational Results | Display/Print/Save Text... from the main exercise window.

Question 7

Compare your classifications against the notes given at the end of the activity.

This part of the activity has been quite involved, but you now should be able to classify any main sequence star according to its spectral characteristics. If you are in contact with anyone else doing this activity you may also have found that there is a certain degree of subjectivity to the classification, but that differences of opinion should be relatively minor.

This would be a suitable point to take a break from this activity. (Again, if you have not done so, remember to save the file of your results using the menu item Tools | Results Editors | Observational Results | Save Data... from the main exercise window.)

Using the telescope simulator to obtain stellar spectra

Background

This part of the activity will introduce you to an important practical aspect of stellar spectrometry. You will investigate how the choice of telescope and the time spent collecting data affect the quality of the spectrum obtained. As you will see, the quality of the spectrum limits an astronomer's ability to assign a spectral classification.

The VIREO program provides a choice of three (simulated) telescopes. Large modern telescopes are extremely expensive resources. Many are constructed and managed by international collaborations. This severely limits the individual astronomer's access to telescopes and forces the astronomer to make best use of the instruments available.

A large proportion of a telescope time is usually devoted to obtaining spectra. A spectrometer is attached to the telescope. The starlight from the telescope is passed into the spectrometer via a narrow opening, or slit. Inside the spectrometer a diffraction grating spreads the starlight out according to its wavelength. This spectrum is detected across a row of electronic detectors. Each individual detector collects photons within a narrow range of wavelengths. These narrow ranges are called 'channels'.

The number of photons arriving in each channel per second will be small. This is because the stars are faint, and therefore few photons arrive at the telescope, and because only a small fraction of these photons will fall within any individual channel.

You will be working with a simulation of a photon-counting spectrometer, which simply counts the number of photons received in each channel during the observation period (the integration or exposure time). Unfortunately, photons in any channel arrive randomly. This is rather similar to a familiar problem with buses. The timetable may predict, say, six buses past a particular stop during a 30 minute period, but you might count only four, or perhaps seven. Just as the bus service irregularities cause problems for passengers, so the irregularities in the arrival times of photons cause problems for astronomers.

The difficulty is that each time we measure a particular star's spectrum for the same length of time, we will count different numbers of photons even in the same individual channel. This variability is termed 'noise'.

The noisiness of the data is assessed in terms of the 'signal-to-noise ratio' (often denoted 'S/N'). For a given telescope and measurement period this is essentially the number of signal photons (equivalent to the number of buses predicted from the timetable) divided by the typical variation in the number of photons received. A low signal-to-noise ratio, for instance less than 10, means that features in the spectrum are being obscured by noise. When S/N has a numerical value of one the noise is as large as the signal and any feature seen in the spectrum might in fact be wholly due to random noise. So a spectrum with $S/N = 1$ would be useless – we could not infer anything from it.

For any source, the signal-to-noise ratio can only be improved by increasing the number of photons that constitute the measurement.

- ■ In observing a particular star, what are the two ways in which the number of photons within an individual channel of a spectrometer could be increased?

- ❑ Either the measurement could be made over a longer time interval or a telescope with a larger collecting area could be used.

Since the second option – that of using a larger telescope – is expensive, astronomers usually adopt a compromise between these two factors.

Astronomical telescopes are usually very large reflecting telescopes, and are rated by the size of their primary mirrors. A 1.0 metre telescope has a primary mirror 1.0 metre in diameter and this mirror is the photon collection surface. Attaching a spectrometer to a larger telescope is analogous to inserting a larger funnel to a rain gauge – you collect a lot more light (or rain water).

Using the CLEA stellar spectra exercise you will assess the signal-to-noise ratio obtained with three different telescopes. These are 0.4 m, 1.0 m and 4.0 m telescopes. The 4.0 m telescope may have a mirror bigger than the room you are working in! (Note that the observatory software sometimes refers to the 1.0 m telescope as 0.9 m.)

Part 3 Making observations with different telescopes

This part of the activity should take about 45 minutes.

If you are continuing directly from the classification activity above, close the Classify Spectra window by clicking the X at the top right corner. If you are starting this part of the activity from the main opening screen (after log-in) of the software package, then select File | Run Exercise | "Classification of Stellar Spectra". You will now see the screen titled 'VIREO Exercise – The Classification of Stellar Spectra'.

- Start the observation exercise by accessing a telescope. Select Telescopes | Optical | Access 0.4 Meter. A pop-up window will tell you when you have been given control of the telescope. Click OK.

You will now see a simulated telescope control panel, with a monitor that can show the view through the telescope. At present the telescope is not in use, and the control panel monitor shows the dome interior and shutter in the red lighting that is used to preserve night vision within the observatory.

- Click the Open switch to start the night's work. Once the dome is open, click the button currently labelled 'Off' to access the telescope control panel. This will open another window. Spend a few moments enjoying the view through the telescope's small finder telescope. Note the stars being carried through the field of view by the Earth's rotation. Notice the celestial coordinates, right ascension and declination, with the right ascension changing because of the Earth's rotation (see Section 1.2.1 of the *Observational activities* booklet). Check the date and time on the left-hand side of the window.
- To take control of the telescope click Tracking. This causes the telescope to very slowly rotate about an axis parallel to the Earth's rotation axis, compensating for the Earth's rotation. The stars now seem to stand still. This is necessary for you to be able to collect light from the star in your spectrometer.

You will first of all make measurements on a bright star in the Pleiades star cluster.

- The telescope can be pointed at a predefined position on the sky by selecting the menu item Slew | Set Coordinates…

- In the box that appears, enter the coordinates ‘R.A. 3h 48m 21s’ and declination ‘23° 25′ 17″’, check that the Epoch of Input Coordinates is set to J2000 and press OK. This will place the star HD 23753, one of the brightest in the cluster, near the centre of the field of view of the finder.
- Slide the View control from Finder to Telescope to switch the monitor from the finder to the 0.4 m telescope.

Your star will now be near a pair of red lines. These lines represent the sides of the slit entrance to the telescope’s spectrometer. When the light is centred between the lines, starlight will pass through the slit into the spectrometer.

- Centre the star on the slit using the N, S, E, W buttons. The rate of motion can be adjusted using the Slew Rate control.
- To use the spectrometer, first make sure that Spectrometer is selected in the Instrument panel and then click Access. A new window opens to give a graphical display of the spectrum as it is being recorded. You will be able to watch the spectrum being recorded, although the spectrum will not ‘grow’ because the normalized spectrum is displayed.
- Click the Go button to start collecting data. The spectrometer is set to stop counting once the signal-to-noise ratio reaches a value of 1000. This is a high value of S/N and is more than sufficient for any analysis that you will carry out later in the activity. Often it is not possible to obtain this value of S/N when astronomers are observing faint objects.

For all the stars that you measure in this activity it is good practice to record your observations in a systematic way. So, for instance, for every star you should have a record of the following details in your notes.

the telescope used

date and time of observation

star name

right ascension

declination

apparent magnitude of star

integration time/s

total photon count

average photon count per channel

signal-to-noise ratio

Question 8

For your observation of HD 23753 note all the details listed above (note that the VIREO catalogue number for this star is N2230-02207). Watch the spectrum that appears and describe how it changes before the program automatically stops.

The photon count information allows us to work out how many channels this spectrometer has. The total number of photons detected must be equal to the number of channels multiplied by the average number of photons per channel thus the number of channels is the total number of photons divided by the average number per channel. From this, we can see that the spectrometer has 600 channels.

You might want to try running this count again.

- Close the spectrometer window using File | Exit Spectrometer, or by clicking on the X in the corner of the window and then click on Access again to run the count again.
- Make sure that for every observation you take that you record all the observational details in your notes. Note that you should also save the spectrum for future reference by clicking File | Data | Save Spectrum…. Save this spectrum with the default name followed by the letter A (to distinguish the first observation of this star) and make sure that you note down the reference name along with the details of the observation. [Your first file will be stored as e.g. N2230-02207A.SSP]

We are now going to repeat the observation using a telescope that has a 1.0 m diameter mirror. Within the simulator this is sometimes called the '1.0 m telescope' and at other times the 'KPNO 0.9 m telescope' (KPNO stands for Kitt Peak National Observatory – a major facility in the United States). We shall refer to it throughout this activity as the '1.0 m telescope'.

- To access the 1.0 m telescope click Telescopes | Optical | Access 1.0 Meter. The controls of this telescope are identical to those on the 0.4 m telescope that you used earlier.
- Open the Dome, switch on the Tracking, and point the telescope to the same star, using the Slew | Set Coordinates feature, and obtain the spectrum.
- Record the details of this observation in your notes and save the spectrum as N2230-02207B.SSP.
- You should now access a 4.0 m telescope. As before you will need to request time and access the telescope using the Telescopes menu in the main window.

Again, the procedure for operating this telescope is the same as for the smaller telescopes that you have already used. When you have control of the telescope, return to the same star and obtain the spectrum. Save the spectrum as N2230-02207C.SSP.

- What do you notice about the integration time required to reach a signal-to-noise ratio of 1000 between these observations?

❑ The integration time is longest on the 0.4 m telescope, and gets progressively shorter as larger telescopes are used.

Question 9

(a) If a 1.0 m telescope collects 1000 photons in one second, how many photons would you expect a 4.0 m telescope to collect?

(b) If a 4.0 m and a 1.0 m telescope are used to observe the same object, then the larger telescope will reach a specified value of S/N in a shorter time than the smaller telescope. What is the relationship between these integration times on the two telescopes? Are your expectations borne out by the integration times required to reach $S/N = 1000$ in the case of HD 23753?

(c) If you were an astronomer applying for telescope time to measure the spectra of 50 stars that are of similar apparent magnitude to HD 23753, which one of the three telescopes would you ask to use? (Think about the time taken per observation – not just the integration time.)

- Next you will measure the spectrum of a fainter star. This has no common name so we will call it S2. Move the telescope to 3h 44m 56s, 24° 29′ 30″, and centre it in the monitor. The VIREO catalogue name for this star is N2230-00564.
- From the telescope control panel switch the monitor back to the Finder using the View slider. You will see that the star is too faint to be distinguishable in the small finder telescope.
- Switch back to the 4.0 m telescope view and finalize the positioning. Take a reading, but in this case stop when the S/N reaches 100 by clicking Stop Count. As usual, record the details of the observation in your notes. Save the spectrum.

Bearing in mind what you have seen so far, think about the following.

- What do notice about the way in which this spectrum builds up? How does it compare to the previous spectra that you have measured?

❑ The early measurements may scarcely look like a spectrum. The noise is greater, and takes much longer to damp down.

Although the signal-to-noise ratio continually increases, it increases quite slowly. This emphasizes the point that high signal-to-noise measurements for fainter stars may require a long integration time.

Part 4 Classifying your two spectra and working out distances

This part of the activity should take about 45 minutes.

In this part you will classify the stars that you observed in Part 3 of the activity. The absolute magnitudes of main sequence stars are known (see Table 1). Using the apparent magnitudes of these two stars (which you will have noted in your observations) you can calculate the distance of the Pleiades cluster. The technique of determining distance in this way is called spectroscopic parallax.

Table 1 Absolute visual magnitude versus spectral type (from C.W. Allen, *Astrophysical Quantities*, The Athlone Press, London, 1973).

Spectral type	Absolute magnitude, M_V
O5	–5.8
B0	–4.1
B5	–1.1
A0	0.7
A5	2
F0	2.6
F5	3.4
G0	4.4
G5	5.1
K0	5.9
K5	7.3
M0	9
M5	11.8
M8	16

The first step is to classify the two stars.

- Switch back to using the spectral classification part of the software package.

- From the main window select the menu item Tools | Spectral Classification.
- Once you have the Classify Spectra window open you need to load the spectra that you took at the telescope. Select the menu item File | Unknown Spectra| Saved Spectra.

This will open a list of all your saved spectra. The filenames have the following format,

```
FILENAME.SSP
```

- Select the spectrum of HD 23753 taken with the 4m telescope, which will be called N2230-02207C.

Now use the Atlas of main sequence spectra to identify this spectrum as you did for the unknown spectra earlier. When you have classified this spectrum, repeat your analysis for the star S2

Question 10

(a) What are the spectral classes of HD 23753 and S2?

(b) From Table 1, what are the absolute magnitudes of these two stars? (If the absolute magnitude of the spectral class is not listed in Table 1 then you should consider how to make a reasonable estimate given the tabulated data.)

The relationship that links absolute magnitude M, apparent magnitude m and distance d is

$$M = m - 5\log(d/\text{pc}) + 5$$

(*An Introduction to the Sun and Stars*, Equation 3.16)

This can be rearranged to obtain an equation for the distance

$$d/\text{pc} = 10^{0.2(m-M+5)} \qquad (1)$$

You may recall that absolute magnitude is equal to the apparent magnitude that the star would have if it was at a distance of 10 pc from Earth.

Now let us calculate the distance to HD 23753. The apparent magnitude is $m = 5.44$. The absolute magnitude (see the answer to Question 10) is –0.74. Substituting these values in to Equation 1, gives

$$d/\text{pc} = 10^{0.2(5.44-(-0.74)+5)} = 10^{2.236} = 172 \text{ pc}$$

So the technique of spectroscopic parallax yields a distance to HD 23753 of 172 pc.

Question 11

Using the absolute magnitude that you determined in Question 10 and the apparent magnitude that you measured, calculate the distance to S2. Compare this result to that obtained for HD 23753.

As you found in your answer to Question 11 there is a wide variation between individual measurements of distance made using this technique. You can see from Table 1 that a difference in assigned spectral type of just one or two subclasses can cause a change in absolute magnitude of half a magnitude or more, which results in relatively large uncertainties in the distances calculated using spectroscopic parallax technique. In fact, if you had been unlucky it is possible

that one (or both!) stars may not have been members of the Pleiades at all, but stars which lie either in front of or behind the cluster. One way of overcoming these problems, at least as far as determining the distance to the Pleiades is concerned, is to make measurements of several stars and then to find their mean (or average) distance. This is the final task of the activity.

Question 12

Table 2 gives the coordinates of six stars in the Pleiades. You have already measured the distance to two of these stars. Using the technique of spectroscopic parallax, measure the distance to the remaining four stars. Hence estimate the distance to the Pleiades by calculating the mean distance to all six stars.

Table 2 The coordinates of the six target stars in the Pleiades.

Star name	VIREO catalogue name	Right ascension (J2000)	Declination
HD 23753	N2230-02207	3h 48m 21s	23° 25′ 17″
S2	N2230=00564	3h 44m 56s	24° 29′ 30″
HD 23733	N2230-01546	3h 48m 14s	24° 19′ 06 ″
S4	N2230-00974	3h 45m 35s	24° 05′ 00″
S5	N2230-00863	3h 49m 39s	24° 13′ 38″
HD 23713	N2230-01632	3h 48m 07s	24° 08′ 32″

Endnote

In this activity you have studied main sequence stellar spectra and learnt to use the Harvard Spectral Classification scheme. You have also noticed the effects of noise on measurements, and tested the options of extending observation time or moving to a larger telescope to improve the signal-to-noise ratio. Finally you have used the telescope simulator to find a distance to the Pleiades cluster using the technique of spectroscopic parallax.

Answers to questions

Question 1

From your analysis of the spectrum of HD 124320 you should have found:

(a) The normalized intensities are 0.97 at 4050 Å and 0.76 at 4400 Å, so that the longer wavelength has about three-quarters of the intensity of the shorter wavelength.

(b) The continuum seems to have a maximum value at around 4040 Å, although this is not very clear because of the deep and wide spectral lines in this region.

(c) The fact that the continuum seems to peak at around 4000 Å and decreases at longer wavelengths implies that the spectral flux density has a maximum in the violet or blue part of the spectrum, and suggests the star will appear blue.

(d) The blue colour would be indicative of a photosphere that is hotter than that of the Sun.

Question 2

The strength of spectral lines varies through the sequence of spectral classes because the strength of lines varies with photospheric temperature. This dependence arises because the number of atoms (or ions) that are in the particular electronic state that can give rise to a line is strongly dependent on temperature. For instance, the Hγ line arises when a photon is absorbed by a hydrogen atom in the $n = 2$ state. At low temperatures hydrogen is predominantly in the ground state ($n = 1$), and so the Hγ line will be weak. At high temperatures hydrogen will be excited to above the $n = 2$ state or may be ionized, and again the Hγ line will be weak. At intermediate temperatures there will be relatively large numbers of hydrogen atoms in the $n = 2$ state and the Hγ line will be strong.

Question 3

Using the Show Difference function, A1 is now easily the closest, with A5 quite close. So we can say that HD 124320 is between A1 and A5, and closer to A1, so class A2 is appropriate.

Question 4

The wavelength is approximately 3935 Å and its value is 0.14. This small peak in the difference indicates that the absorption line here is deeper in HD 124320 than in the reference. From the table, the standard wavelength of the CaII K line is 3933.68 Å. The 'II' means the calcium atoms producing the line are singly ionized. They are each missing one electron, because of the high temperature of their environment. (Note that the calcium II spectrum has two close lines at the visible/UV border. The other, the CaII H line, is at 3968 Å.)

Question 5

The CaII K line is visible throughout almost the entire range of reference spectra provided. The K line is rather shallow in O5, B0 and B6 (the hottest stars), and is hard to detect in M0 and M5 (the coolest stars). Figure 3.23 of Chapter 3 shows that the line strengths of ionized calcium drop off in both early and late spectral types, which corresponds with our findings.

Question 6

(a) The spectrum appears to be midway between B0 and B6, and so a choice of spectral type of B3 would be appropriate. Making a classification to the nearest tenth of a class is quite difficult using a limited selection of reference spectra, so it would be hard rule out the possibility that the star is class B2 or B4. So if you decided that HD 37767 is class B2, B3 or B4, then you would be in agreement with most astronomers. In fact, the Henry Draper catalogue classifies it as a type B3 star.

(b) In your inspection of the spectrum of HD 37767 you should have found many of the lines listed in Table 3. Note that the measured wavelength column shows typical results – don't be concerned if your measurements differ by up to an ångstrom or so.

Table 3 Lines observed in the spectrum of HD 37767.

Measured wavelength/Å	Identification
3969.2	CaII (H line) – 3968.49 Å or HI (Hε) – 3970.07 Å.
4008.2	unknown
4025.3	HeI – 4026.19 Å
4100.9	HeII – 4100.04 or HI (Hδ) – 4101.75
4143.6	FeI – 4143.88
4340.0	HI (Hγ) – 4340.48
4387.2	HeI – 4387.93
4470.9	HeI – 4471.38

Notice that one line is unidentified, suggesting that it is not common enough to merit inclusion in the software's files. Every star is an individual, and spectral classification is about grouping similar, but not identical, stars.

There are also some ambiguous identifications. The line at 3969.2 Å may be the CaII H line or Hε. In fact, because the CaII K line is very weak in this spectrum and both the CaII H and K lines tend to occur with similar strengths, it is actually much more likely to be Hε. The line at 4100.9 Å is actually the Hδ line rather than the HeII line.

Question 7

The classifications of the stars HD 6111 and HD 5351 are described in detail below. A full reference list of all stars in the Unknown spectrum list is given in Table 4. In general, you should be able to classify most of these stars to within two-tenths of the spectral class listed here (i.e. a spectral class subdivision that is ±2 from the tabulated value). Note that HD 35215 is classed as a B1.5 – you should be satisfied if you obtained B1 or B2. The one star that might cause you problems is SAO 81292 – this has the spectrum of an M4 or M5 star (in fact it is between these two) but shows emission lines that are not seen in any of the reference spectra. These peculiar emission features are denoted in the classification scheme by the letter 'e'.

Table 4 The spectral classes of the stars in the Unknown spectrum list.

Star	Class	Star	Class	Star	Class
HD 124320	A2	BD+63 137	M1	HD 242936	O8
HD 37767	B3	HD 66171	G2	HD 5351	K4
HD 35619	O7	HZ 948	F3	SAO 81292	M4.5e
HD 23733	A9	HD 35215	B1.5	HD 27685	G7
O 1015	B8	Feige 40	B4	HD 21619	A6
HD 24189	F6	Feige 41	A1	HD 23511	F4
HD 107399	F9	HD 6111	F8	HD 158659	B0
HD 240344	B4	HD 23863	A7		
HD 17647	G1	HD 221741	A3		

HD 6111

The classes F5, G0 and G6 all look very similar to the spectrum of HD 6111. The most prominent lines are the strong CaII (K and H) lines. There are three weaker lines: HeII, HI (Hγ) and the CH/metals band. Look closely at the difference display for each of these. The G0 differences for these lines are all very close to zero, so G0 is the closest reference class for HD 6111. In fact, The catalogue lists the star as F8. This is quite similar to our own Sun. You might like to switch to the greyscale display and compare this to the solar spectrum in Figure 1.28 of *An Introduction to the Sun and Stars*.

HD 5351

For HD 5351, K5, M0 and M5 are all close. By studying the difference displays for each of the prominent lines, the CaI line at 4226.74 Å seems quite sensitive. You can also assess the CaII H and K lines, and the MnI line at 4030.70 Å. The catalogue classification is K4, though you might think it could reasonably be placed a few tenths later.

Question 8

You should have a table of observational details that resembles that given in Table 5. (There will be small differences in measured quantities between your results and those quoted here because of observational uncertainties.)

Table 5 Details of a single observation (see Question 8).

telescope	0.4 m
date and time	22/09/02 14:30
star	HD 23753
right ascension (J2000)	3h 48m 21s
Declination (J2000)	23° 25′ 17″
apparent visual magnitude	5.44
integration time/s	102.5
total photon count	600 043 253
average photon count per channel	1000 073
signal-to-noise ratio	1000

As the spectrum was being measured you should have noticed that the individual data points jitter up and down. The 'jitteriness' is the noise, due to random arrival times of the photons. As you watch, the variations slowly die down. This means that in the course of the measurement the signal-to-noise ratio has improved (increased), which you may have noticed by watching the spectrometer signal-to-noise readout. A longer measurement time reduces the noise, giving a more reliable spectrum. 'Integration' is simply the time taken for the measurement.

Question 9

(a) The number of photons collected will depend on the collecting area. The ratio of primary mirror diameters is 4 : 1 so the ratio of areas is $4^2 : 1^2$ or 16 : 1. The larger telescope will collect 16 times more photons, giving 16 000 photons in one second.

(b) Since the same total number of photons is required for the given *S*/*N*, the larger telescope reaches the required level 16 times faster.

(c) The sensible rule of thumb for applying for telescope time is to apply for the smallest diameter telescope that can be practicably used. We've already seen that the 0.4 m telescope can be used to measure the spectra of these stars, so clearly it looks as though it could be appropriate. However, let us work out how much observing time is needed on the 0.4 and 1.0 m telescopes to carry out all 50 observations.

The 0.4 m telescope requires an integration time of about 20 seconds per star, and the total integration time for the observations would be about 1000 seconds or about 17 minutes. On the 1.0 m telescope the integration time is about 3 seconds per star, or roughly about 3 minutes in total. However, the observation time includes time taken to move a telescope – as you have seen it would probably take at least a minute to change the pointing of the telescope from one star to another, so positioning any telescope would take 50 minutes. So on a 0.4 m telescope your observations would take 67 minutes, whereas on a 1.0 m telescope the observations would take 53 minutes. So, in this case there is no great advantage in using a larger telescope – the 0.4 m telescope would be sufficient – and an application to use the 1.0 m telescope is likely to be soundly rejected!

Question 10

(a) The spectral class of HD 23753 appears to be almost identical to the reference spectrum B6, and is therefore classed as B6. Star S2 is very similar to spectral type M0 but not identical. Comparison with the K5 and M5 spectra indicates that it may be slightly closer to K5 so a spectral type of K9 is assigned.

(b) HD 23753 has been assigned a spectral type of B6. The absolute magnitude of a type B6 star is not listed, but presumably lies between that of B5 ($M = -1.1$) and A0 ($M = 0.7$). An estimate of the absolute magnitude can be made in the following way. Spectral type B6 is 1 spectral subdivision beyond B5, and A0 is 5 spectral subdivisions beyond B5, and so the absolute magnitude of B6 could be estimated by taking the absolute magnitude of B5 and adding 1/5 of the difference between the absolute magnitudes of B5 and A0.

$$M(\mathrm{B6}) = M(\mathrm{B5}) + 1/5 \times (M(\mathrm{A0}) - M(\mathrm{B5}))$$

$$M(\mathrm{B6}) = -1.1 + 1/5 \times (0.7 - (-1.1)\ ) = -1.1 + 0.36 = -0.74$$

So, an estimate of the absolute magnitude of a B6 main sequence star is $M = -0.74$. (In fact, this only provides a crude estimate of the absolute magnitude, but one that is adequate for this calculation.)

Star S2 has been assigned a spectral type of K9. Again a rough estimate of absolute magnitude can be made in a similar way as was done for a type B6 star,

$$M(\mathrm{K9}) = M(\mathrm{K5}) + 4/5 \times (M(\mathrm{M0}) - M(\mathrm{K5}))$$

Using the values from Table 1,

$$M(\mathrm{K9}) = 7.3 + 4/5 \times (9 - 7.3) = 8.66$$

So, an estimate of the absolute magnitude of a main sequence star of type K9 is $M = 8.66$.

Question 11

The distance to star S2 is found using values for apparent magnitude of $m = 14.25$ and absolute magnitude (see the answer to Question 10) of 8.66. Substituting these values in to Equation 1, gives

$$d/\text{pc} = 10^{0.2(14.25-8.66)+5)} = 10^{2.118} = 131 \text{ pc}$$

So the technique of spectroscopic parallax yields a distance to star S2 of 131 pc.

This is quite different to the distance derived for HD 23753 (172 pc). The two stars do lie within the same cluster. Unfortunately, when 10 is raised to a power of a number, a small uncertainty in that number translates into a large uncertainty in the result. So minor uncertainties in the absolute or apparent magnitudes can lead to large uncertainties in distances.

Question 12

The results of the distance estimates to all six stars are shown in Table 6. Note that your results should be similar, but may not be identical to those shown here.

The mean distance of these stars is 130 pc, which agrees well with the currently accepted distance to the Pleiades of about 135pc.

Table 6 Results for the distance measurements for stars in the Pleiades.

Star name	Spectral type	M	m	$M - m$	distance/pc
HD 23753	B6	–0.74	5.44	6.18	172
S2	K9	8.66	14.25	5.59	131
HD 23733	F0	2.60	8.27	5.67	136
S4	G3	4.82	10.11	5.29	114
S5	K2	6.46	11.20	4.74	89
HD 23713	F6	3.60	9.25	5.65	135

Acknowledgement

The activity is based on a software package developed by a group called Contemporary Laboratory Experiences in Astronomy (CLEA) that is based at Gettysburg College, Pennsylvania, USA.

S282 Astronomy

The age of the Jewel Box

Study time: 2 hours

Summary

In this activity you will determine the approximate age of the Jewel Box star cluster by plotting a Hertzsprung–Russell (H–R) diagram. You will plot the positions of stars in the cluster by using a colour image to estimate their spectral class (from their colour) and relative brightness (from the image size of the stars).

You should be familiar with the H–R diagram (Section 4.2 of *An Introduction to the Sun and Stars*) and the properties of clusters of stars (Section 3.2.4 and 4.2.5) before attempting this activity.

Learning outcomes

- Be able to plot the stars of a cluster on an H–R diagram and estimate the age of the cluster.
- Appreciate the difficulties of working with real data.

Preparation

You will need:

- The printed colour plate showing the Jewel Box cluster and the colour/brightness gauge (supplied within your course mailing).
- The blank H–R diagram worksheet (which also provides sample cluster H–R diagrams in the same form). This worksheet is supplied at the back of this activity.
- Scissors, ruler and a pencil.
- A hand lens, a clear (not frosted) plastic A4 pocket and a washable marker might be useful.

This activity is best done in reasonably bright light, daylight if possible.

Note: if you have poor colour vision you may find you are unable to do this activity without help.

Background to the activity

The Jewel Box is an open cluster in the constellation Crux, the Southern Cross. It looks like a star to the unaided eye, but is only visible from the southern hemisphere. The cluster contains just over 100 stars, spans about 6 pc and lies about 2300 pc away. It was named the Jewel Box from its appearance in the telescope, which was described by Sir John Herschel as ‘a casket of variously coloured precious stones’.

In this image, as in most images of stars, the photographic process means that the brighter the star the bigger it appears.

In this activity you will be drawing a simple H–R diagram of the Jewel Box cluster in the form of a colour–magnitude diagram (see page 133 of *An Introduction to the Sun and Stars*). These are usually constructed from observations using images in two filters (blue, B, and visual, V) from which the V magnitude (which is related to luminosity *if* all the stars are at the same distance) and colour (in the form of colour index $B - V$ which is indicative of temperature) are derived. You will use the size of the stellar images as a measure of brightness (V) and the perceived colour of the star as a measure of the colour index ($B - V$).

Question 1

From your study of Section 4.2 of *An Introduction to the Sun and Stars*, explain why it is possible to estimate the age of a star cluster by plotting its stars on an H–R diagram.

The activity

- Carefully cut the strip containing the colour/brightness gauge away from the Jewel Box image. If you put the image into a plastic pocket then it is easy to mark what you have done without messing up the image, but you may find the pocket makes the colours difficult to see.

Examine the image when you have done this.

- Do all the stars appear to be the same colour?

❑ The less bright, smaller stars nearly all show distinct colouring. The brighter ones tend to appear white in the centre, but most of them have a rim of colour around the limb. The white centres are due to overexposure of the photographic plate. (This is the same factor that makes the brighter stars appear so much larger than the dimmer ones. They are not really noticeably bigger at this distance, and certainly not as big as they appear in images like this.)

Question 2

Can you tell where the edge of the cluster lies?

- Using pencil if you are working directly on the image, or marker if using the plastic pocket, outline where you think the boundaries of the cluster are.
- Place an X where you estimate the centre of the cluster of stars to be and use a ruler to draw a 4 cm square about this centre point. Measure the brightness of the star closest to the upper left-hand corner of your square from its size in the image in comparison to the dots on the colour/brightness gauge. Estimate the star's colour using the colour portion of the colour/brightness gauge and mark a dot on the blank H–R diagram in the box that corresponds to the brightness and colour you have measured for your first star.
- Mark the star you have just measured and then proceed in some systematic fashion to measure the brightness and colour of every star within your 4 cm square. (Remember that the colours of the brighter stars are only apparent around their edges where the image is not overexposed.)

Question 3

Do the Jewel Box stars on your diagram appear to be randomly scattered or do they fall in any kind of pattern? What problems have you encountered in measuring the colour and brightness of the individual stars?

Stars in front of or behind the Jewel Box that are not part of the cluster also appear in the image. These are known as 'field stars'. If time allows, estimate how many of these stars are included in your measurements by drawing a 4 cm square near the edge of the print and measure the colour and brightness of the stars within this square. Mark these stars on your H–R diagram using another symbol, e.g. × instead of a dot.

Question 4

Do the field stars appear to fall randomly on your diagram or do they appear to fall in any kind of pattern?

Question 5

Compare your answers to Questions 3 and 4. Why do you think the similarities or differences between the two star patterns exist?

Question 6

Using the sample H–R diagrams on the worksheet give a rough estimate of the age of the Jewel Box cluster.

Question 7

Thinking about the relationship between mass, luminosity and colour of main sequence stars, review what the three sample H–R diagrams show about the relative main sequence lifetimes of O/B stars compared to A/F/G stars compared to K/M stars.

Answers to questions

Question 1

A cluster occurs because the stars in it form from the same dense cloud at the same time. This means that the stars in the cluster are the same age and have similar composition, so they vary only in mass. Because stars of different masses evolve at different rates, and we can tell the mass of a main sequence star from its position on the main sequence, this means we can estimate the age of the cluster from the masses of the stars remaining on the main sequence compared to those that have left it.

Question 2

The edge of the cluster is not very clearly defined, but you should have identified a denser area of stars about 8–10 cm across, with the bright orange star a little off centre.

Question 3

You may have been surprised by just how many stars there were in a 4 cm square! Most of the stars should fall in a band stretching up to the left from the bottom right-hand corner of the H–R diagram, from brightness 10, colour K2 or M to about brightness 5 or 6, colour A – this corresponds to the main sequence. There are a few hotter brighter stars in the top left-hand corner, and you may have found some in other areas, including the bright orange star near the centre of the cluster. You may have had problems deciding which colour or brightness category some of the stars fall into, and in particular seeing the colours on some of the less bright stars, and may have rather a lot plotted as spectral class A – the class which appears as white.

Remember that on the H–R diagram the temperature increases to the left, corresponding to shorter wavelengths on the peak of the stars' black-body spectrum, so that hot stars appear blue, with the colour fading through white to yellow then orange and reddish for low temperature stars, corresponding to the spectral classes on the colour/brightness gauge.

Question 4

Generally the field stars will appear to be more randomly spread across the lower half of the diagram.

Question 5

The field stars are at different distances from us than the Jewel Box cluster. Using an image taken from Earth we are looking at apparent brightness, not absolute brightness, so a more distant star will appear less bright than a similar star in the Jewel Box, and will plot lower on the diagram. But bear in mind that there will also be field stars in the same region of the sky as the cluster, and there may be one or two outlying members of the cluster in the region you chose for your field stars.

Question 6

With the exception of the bright orange star almost all the rest plot on the main sequence. There appear to be none plotting on the red giant branch as in the middle aged and old clusters on the worksheet and M67 in Figure 4.10b of *An Introduction to the Sun and Stars*. The cluster therefore is similar to the young cluster in Figure 4.10a, so we can conclude that it is less than 100 million years old.

In fact the Jewel Box is only about 12 million years old. The bright orange star is known as κ (kappa) Crucis. It is a very large (hence very luminous) star which, though quite young in stellar terms, has already moved into a red supergiant phase.

Question 7

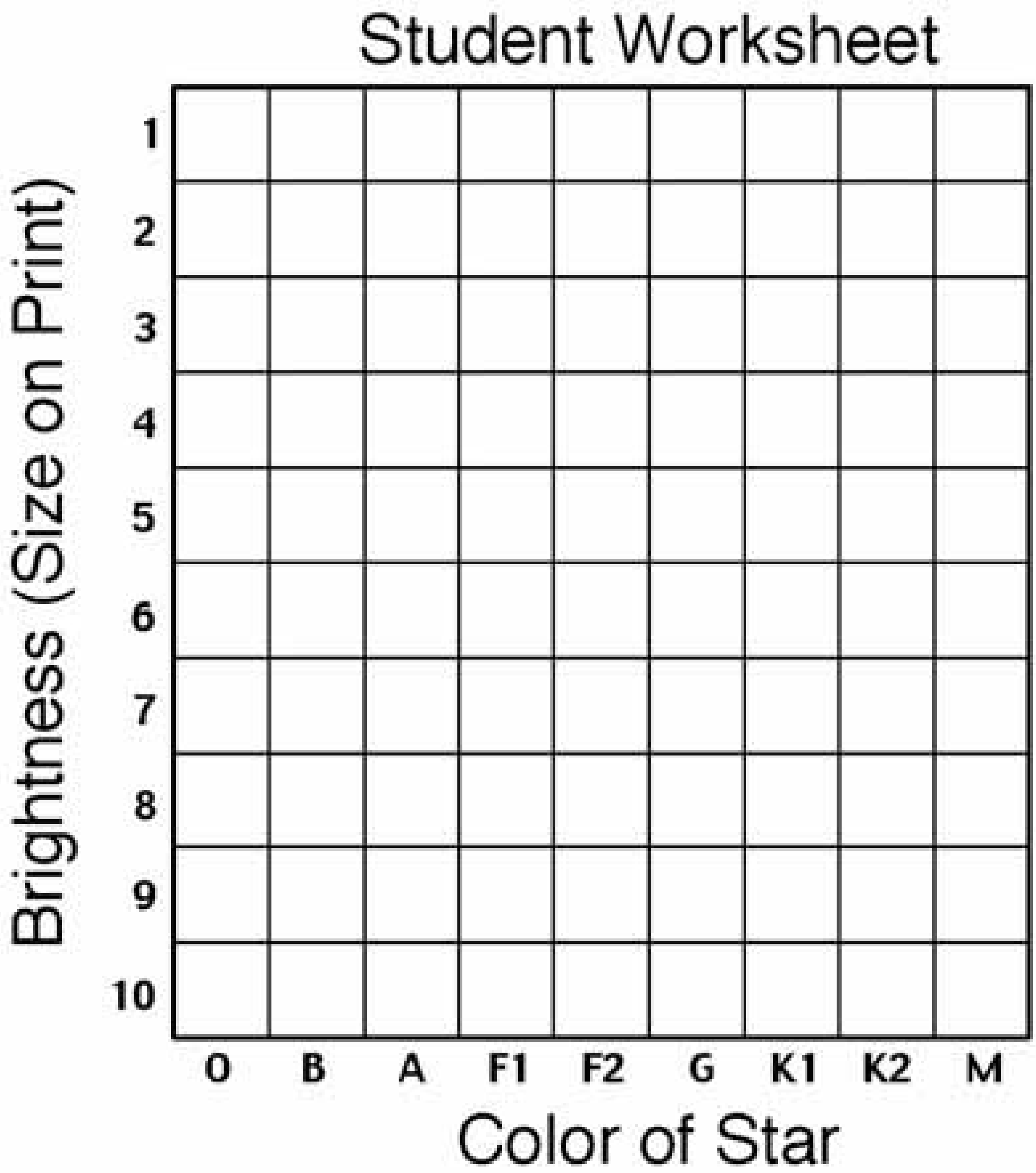
The more massive a star the hotter and brighter it is, and so it plots higher up the main sequence. More massive stars also burn the hydrogen in their cores faster, and so have shorter main sequence lifetimes. Very massive stars spend only of the order of 10 million years on the main sequence and then become supergiants with luminosity more than 10^4 times that of the Sun. As lower mass stars leave the main sequence they become red giants, which are cooler but more luminous than main sequence stars, and plot in a distinct region above and to the right of the main sequence on the H–R diagram.

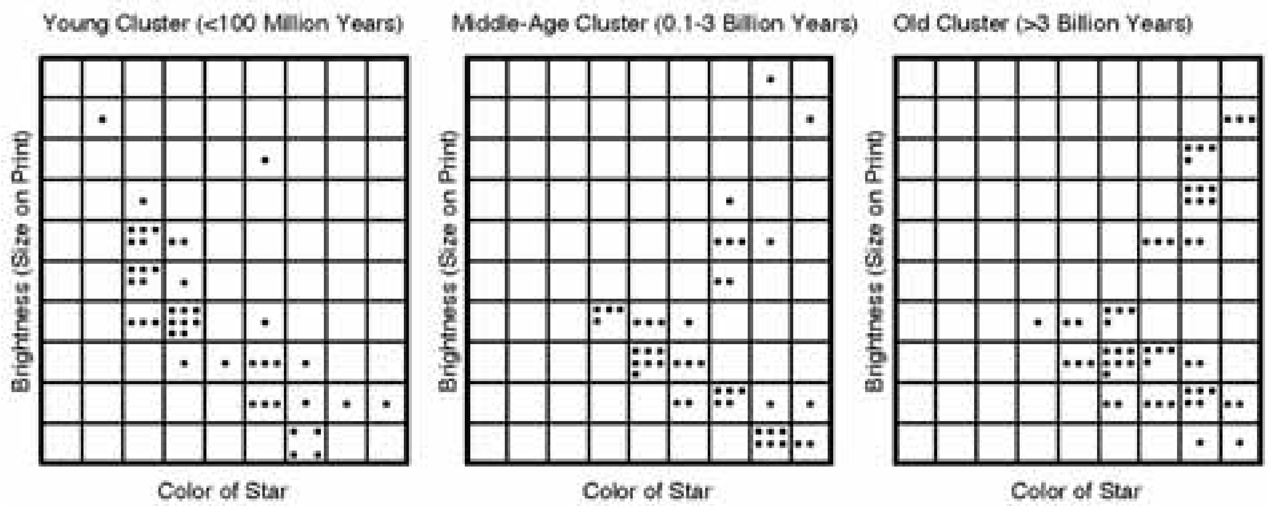
As a cluster ages the stars in it reach the end of their main sequence lives, so if you plot the cluster stars on an H–R diagram as you have here you can see which are the hottest and brightest stars which remain on the main sequence – the turn-off point – and hence estimate the age of the cluster. The sample H–R diagrams show that the massive O/B stars are very short-lived since they have moved away from the main sequence even in the first 100 million years. By 3 billion years many A/F/G stars have completed their main sequence lives, but the low mass K/M stars still remain on the main sequence.

Acknowledgements

Worksheet: National Optical Astronomy Observatory

Colour plate: Nick Suntzeff (CTIO)/NOAO/AURA/NSF





Worksheet: National Optical Astronomy Observatory

Jeans mass

Study time: 90 minutes

Summary

In this spreadsheet activity you will create a formula to determine the Jeans mass of interstellar clouds. You will then use it to investigate the influence of the different cloud properties on the tendency of the cloud to contract.

You should have read Chapter 5 of *An Introduction to the Sun and Stars* before attempting this activity.

Learning outcomes

- Gain an awareness of the range of conditions under which a uniform spherical interstellar cloud is likely to collapse.
- Create a complex formula in a spreadsheet and use it to perform calculations.

Background to the activity

If the mass of a uniform spherical cloud exceeds the Jeans mass the force of gravitational attraction will overcome the opposing pressure due to the motion of the particles, and contraction will occur. This critical mass depends on the density, temperature and composition of the cloud, and is given by the expression:

$$M_\mathrm{J} = \frac{9}{4} \times \left(\frac{1}{2\pi n}\right)^{1/2} \times \frac{1}{m^2} \times \left(\frac{kT}{G}\right)^{3/2} \qquad (1)$$

(*An Introduction to the Sun and Stars*, Equation 5.1)

Although this equation provides a very simplified criterion for the collapse of an interstellar cloud it does provide a useful guide to the conditions under which that collapse is likely to occur. You will not consider here the complex aspects of cloud collapse such as inhomogeneities in properties of the cloud, rotation or magnetic fields. You will use the Jeans mass as a tool in investigate the conditions under which an ideal cloud is likely to collapse and to develop further your spreadsheet skills in use of formulae.

Part 1 The Jeans mass spreadsheet

The instructions given here assume that you will be using the StarOffice package that is supplied with the course and that you have completed some of the earlier spreadsheet activities. If you have not completed any other spreadsheet activities you will probably need to consult the *Using Spreadsheets* guide on the course website for instructions on some of the basic procedures not described in this activity, such as formatting cells.

If you are already familiar with using another spreadsheet package (such as Microsoft Excel) you may want to use that to carry out the activity. However, before starting, you should be aware that these notes only give instructions on how to manipulate the StarOffice spreadsheet.

Set up the spreadsheet

- If you have not already done so, start StarOffice. From the main StarOffice menu select File | New | Spreadsheet to create a new, blank sheet.

Don't forget to save your work regularly in your work folder. (You should already have set up such a folder for earlier activities. If not, create a work folder now.) From time to time make a backup copy of your work (using a different filename) in case you need to go back to an earlier stage.

As in the earlier activities, consider carefully the layout of your spreadsheet. In this activity you will construct a complex formula stage by stage, so it is vital that you include sufficient *labelling* and other information to allow any other user of the spreadsheet (or yourself if you return to the spreadsheet after some time) to understand what is going on. (You may wish to refer back to the formatting that you did in earlier activities as a reminder.)

The Jeans mass formula

In order to use the formula in a spreadsheet it is sensible to break it down into sections containing all the constants (which will be fixed in all calculations) and the variable quantities.

Question 1

For each symbol in the Jeans mass equation write down: its meaning, whether it is a constant or a variable, its value (if a constant) and its SI units.

- Enter a section in your spreadsheet with headings 'Constant' 'Symbol' 'Value' 'Unit' and insert the appropriate constants from the Jeans mass equation. To enter the value of π in the spreadsheet, use the function `=PI()`.
- Format the cells appropriately (scientific notation is required for most of the numbers).

Your spreadsheet should look like Figure 2 in Note 1 (see the 'Notes' section towards the back of this activity).

Question 2

Rearrange the Jeans mass equation so that all the constants are separated from the variables.

- Insert a new constant in your spreadsheet table (you can call it A), which is the combined constants you defined in the answer to Question 1. You can calculate the value of this constant using a calculator or you can enter it as a formula.

Question 3

What is the value of A? What are the units of A?

If you want to enter A as a formula you need to remember the basic rules for arithmetic. The order for arithmetic operations to be performed is:

^ (to the power of)

× and / (multiply and divide)

+ and – (add and subtract)

For example, when entering a number such as $5^{2/5}$ you will get the wrong answer if you enter `=5^2/5` since this represents $5^2/5$. The correct formula is `=5^(2/5)`. (Alternatively, you could write `=5^0.4`). If in doubt always use brackets to separate different parts of a formula and check your answers.

Question 4

What is the spreadsheet formula for the constant A?

Before you enter the formula for the Jeans mass equation you need to set up cells containing the variables.

- Below your list of constants make a new small table with similar headings for the variables in the Jeans mass equation. It is worth highlighting the 'Value' cells in a different colour since this is where you will want to enter some numbers. (See Figure 3 in Note 2.)

So far you have entered all numbers in SI units. It always makes sense to perform all calculations in SI units so that there is no confusion. However, you may want to enter numbers (or answers) in more convenient, non-SI units. Examples are molecular masses in units of the mass of a hydrogen atom (m_H), (or the Jeans mass in terms of solar mass ($M_\odot$)).

- Enter additional columns in your table showing the value and units of quantities in non-SI units.
- Enter formulae in the appropriate cells to convert them into SI units.

Question 5

What is the formula you would enter into the spreadsheet for the Jeans mass?

- Now enter the Jeans mass formula into your spreadsheet. (If you get an error message `Err.503` this may be because you have forgotten to enter values in the cells for the variables.)
- Format the cell to display the result in scientific notation. It is also a good idea to colour highlight the cell containing the result. You should use a colour that is distinguishable from one you used previously to highlight the 'Value' cells.
- Now enter an additional cell to convert the Jeans mass into units of solar mass.

(Your spreadsheet should now look like Figure 4 in Note 3.)

At this stage it is worth testing your results to ensure you have not made an error. With $T = 100$ K, $m = 1m_H$ ($= 1.67 \times 10^{-27}$ kg) and $n = 10^{10}$ m^{-3}, the Jeans mass is 3.03×10^{32} kg or $152.2M_\odot$.

Question 6

Use your spreadsheet to determine if the following clouds are likely to collapse:

(a) A cloud of mass $5M_\odot$ consisting entirely of neutral hydrogen with $T = 30$ K and $n = 10^{11}$ m^{-3}.

(b) A cloud with the same properties as in part (a) but consisting entirely of molecular hydrogen.

Part 2 Alternative versions of the Jeans mass spreadsheet

The spreadsheet you have created allows you to determine the Jeans mass for any set of cloud parameters. However, if you were asked the question: 'To what density would a molecular hydrogen cloud of mass $10M_\odot$ and temperature 20 K need to be compressed before it is likely to collapse?' it would require some trial and error to determine the answer using your spreadsheet.

It is not difficult to modify the spreadsheet to answer this question directly.

Set up a new worksheet

The new version can be entered in a different *worksheet*.

- At the bottom of the spreadsheet you will see a series of tabs marked Sheet1, Sheet2 and Sheet3. The current worksheet will be Sheet1.
- Select Format | Sheet | Rename... from the main menu and rename the sheet 'Jeans mass'. Select the tab Sheet2 and rename it 'Density'.

Since most of the Density worksheet will be the same as the Jeans mass worksheet, it is worthwhile copying the entire contents and editing them.

- Select the entire contents of the worksheet by clicking the square in the corner of the spreadsheet (shown in Figure 1). The worksheet will go dark to show the area selected. Select Edit | Copy from the main menu.

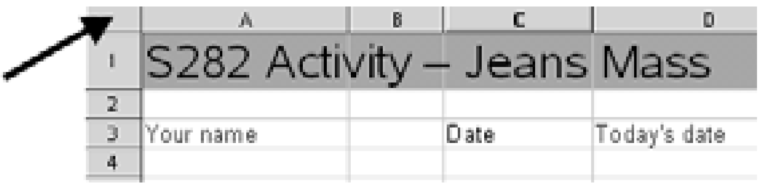


Figure 1 Select this cell to select the whole worksheet.

- Move to the Density worksheet and select the top left corner again. The worksheet will again go dark.
- Select Edit | Paste from the main menu and an exact copy of the Jeans mass worksheet will appear.

The Density worksheet

The only difference required in the new worksheet is that the Jeans mass is to be specified and the density calculated.

- In the *variables* section of your worksheet replace the row containing the density information with the Jeans mass (in SI units) as well as solar masses.
- Make sure you enter the appropriate conversion formula in the *value* cell to convert from solar masses to SI units. Enter an initial value of $1M_\odot$.
- Clear the row containing the Jeans mass formula by selecting the row (click on the row number to highlight the row) then select Edit | Delete contents from the main menu. Click OK in the window that appears.

You will now need to rearrange the Jeans mass equation to obtain the number density.

Question 7

What is the equation for the number density? How would you write this as a spreadsheet formula?

- Enter the formula for the number density in your spreadsheet. (See Figure 5 in Note 4 at the end of this activity.) Test your working using the same data as in the Jeans mass worksheet: $T = 100$ K, $m = 1m_{\mathrm{H}}$ ($= 1.67 \times 10^{-27}$ kg) and $M_{\mathrm{J}} = 3.03 \times 10^{32}$ kg or $152.2M_\odot$ gives $n = 10^{10}$ m^{-3}.

Question 8

Use your spreadsheet to determine the density above which the following clouds are likely to collapse:

(a) A cloud of mass $20M_\odot$ consisting entirely of neutral hydrogen with $T = 15$ K.

(b) A cloud with the same properties as in part (a) but consisting entirely of molecular hydrogen.

Explain why you would expect the result for part (b) to be lower than for part (a).

The Temperature worksheet

In this final part of the activity you will use your experience in setting up the Density worksheet to prepare a third worksheet to answer Question 9.

Note: if you have had difficulty in completing the earlier sections of this activity you may find it more instructive to revise your spreadsheet skills using the earlier spreadsheet activities and the *Using Spreadsheets* guide rather than attempting this part of the activity.

In order to answer Question 9 you will need to:

- Rearrange the Jeans mass equation in terms of the temperature.
- Create a new Temperature worksheet.
- Copy the contents of the Jeans mass worksheet into it.
- Modify the variables and formulae to derive the critical temperature for collapse.

(See Note 5 after you have attempted Question 9.)

Question 9

Use your spreadsheet to determine the temperature below which the following clouds are likely to collapse:

(a) A dense cloud of mass $3M_\odot$ and number density 10^{11} m^{-3} consisting entirely of molecular hydrogen.

(b) A diffuse cloud with the same mass and composition as the dense cloud in part (a) but with a number density of only 10^7 m^{-3}.

(c) A diffuse cloud with the same number density and composition as the diffuse cloud in part (b) but with a mass of $30M_\odot$.

Do the results match your expectations?

Notes

Note 1

When you have set up your Jeans mass spreadsheet it should look similar to Figure 2.

	A	B	C	D
1	S282 Activity – Jeans Mass			
2				
3	Your name		Date	Today's date
4				
5				
6	Constants used in the Jeans mass equation and calculation			
7	Constant	Symbol	Value	Unit
8	Boltzmann const	*k*	1.38E-023	J K^-1
9	Gravitational const	*G*	6.67E-011	N m^2 kg^-2
10	Pi	π	3.14	
11				

Figure 2 The spreadsheet after entering the constants used in the Jeans mass equation.

Note 2

After entering the variables used in the Jeans mass formula the spreadsheet should look something like Figure 3 (variables are entered in cells C15, C17 and E16).

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	*k*	1.38E-023	J K^-1		
9	Gravitational const	*G*	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	*A*	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Number density	*n*	1.00E+010	m^-3		
16	Gas particle mass	*m*	1.67E-027	kg	1	m_H
17	Temperature	*T*	100	K		

Figure 3 The spreadsheet after entering the variables used in the Jeans mass equation.

Cell C16 contains the formula `=E16*1.67E-027`.

Note 3

After entering the Jeans mass formula the spreadsheet should look something like Figure 4 (the shaded cells indicate where variables are entered or key results displayed).

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	*k*	1.38E-023	J K^-1		
9	Gravitational const	*G*	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	*A*	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Number density	*n*	1.00E+010	m^-3		
16	Gas particle mass	*m*	1.67E-027	kg	1	m_H
17	Temperature	*T*	100	K		
18						
19	**Jeans mass**	*M_J*	3.03E+032	kg	152.2	M_sun
20						

Figure 4 The spreadsheet after entering the Jeans mass formula.

Cell E19 contains the formula `=C19/1.99E+30`.

Note 4

Your completed density worksheet should look similar to Figure 5.

	A	B	C	D	E	F
1	S282 Activity – Jeans Mass					
2						
3	Your name		Date	Today's date		
4						
5						
6	**Constants used in the Jeans mass equation and calculation**					
7	**Constant**	**Symbol**	**Value**	**Unit**		
8	Boltzmann const	*k*	1.38E-023	J K^-1		
9	Gravitational const	G	6.67E-011	N m^2 kg^-2		
10	Pi	π	3.14			
11	Combined constant	A	8.45E-020	m^-3/2 K^-3/2 kg^3		
12						
13	**Variables used in the Jeans mass equation and calculation**					
14	**Variable**	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
15	Jeans mass	M_J	3.98E+031	kg	20	M_sun
16	Gas particle mass	*m*	3.34E-027	kg	2	m_H
17	Temperature	*T*	15	K		
18						
19	**Number density**	*n*	1.22E+008	m^-3		

Figure 5 The completed Density worksheet.

Note 5

Rearranging the form of the Jeans mass equation from Question 2:

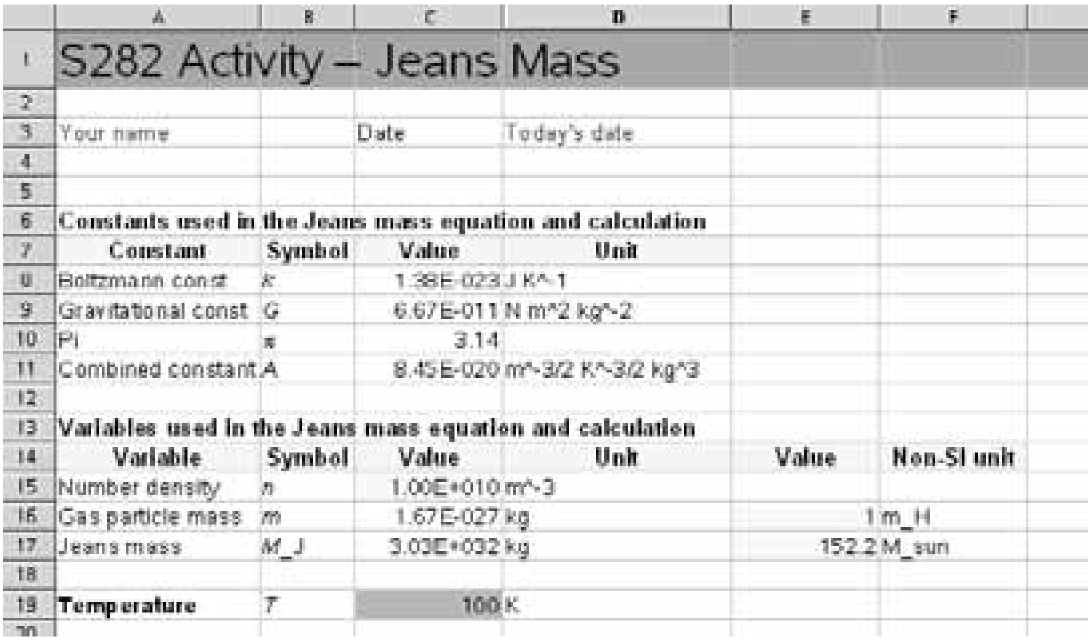
$$M_J = \frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{n^{1/2}m^2}$$

$$\text{gives } T^{3/2} = \frac{4(2\pi)^{1/2}}{9}\left(\frac{G}{k}\right)^{3/2} \times M_J m^2 n^{1/2}$$

$$\text{so } T = \left[\frac{4(2\pi)^{1/2}}{9}\left(\frac{G}{k}\right)^{3/2} \times M_J m^2 n^{1/2}\right]^{2/3} = \left[\left(\frac{1}{A}\right) \times M_J m^2 n^{1/2}\right]^{2/3}$$

The formula used in the temperature spreadsheet (Figure 6) will therefore be

```
=((1/C11)*C17*C16^2*C15^0.5)^(2/3)
```



S282 Activity – Jeans Mass					
Your name		Date	Today's date		
Constants used in the Jeans mass equation and calculation					
Constant	**Symbol**	**Value**	**Unit**		
Boltzmann const	k	1.38E-023	J K^-1		
Gravitational const	G	6.67E-011	N m^2 kg^-2		
Pi	π	3.14			
Combined constant	A	8.45E-020	m^-3/2 K^-3/2 kg^3		
Variables used in the Jeans mass equation and calculation					
Variable	**Symbol**	**Value**	**Unit**	**Value**	**Non-SI unit**
Number density	n	1.00E+010	m^-3		
Gas particle mass	m	1.67E-027	kg	1	m_H
Jeans mass	M_J	3.03E+032	kg	152.2	M_sun
Temperature	T	100	K		

Figure 6 The completed Temperature worksheet.

Answers to questions

Question 1

Table 1 Variables and constants in the Jeans mass equation.

Symbol	Meaning	Variable/constant	Value	SI Units
M_J	Jeans mass	variable		kg
π	the number pi	constant	3.142…	
n	particle number density	variable		m^{-3}
m	mass of the 'average' gas particle in the cloud	variable		kg
k	Boltzmann constant	constant	1.38×10^{-23}	$J\ K^{-1}$
T	temperature	variable		K
G	gravitational constant	constant	6.67×10^{-11}	$N\ m^2\ kg^{-2}$

Question 2

The Jeans mass equation

$$M_J = \frac{9}{4} \times \left(\frac{1}{2\pi n}\right)^{1/2} \times \frac{1}{m^2} \times \left(\frac{kT}{G}\right)^{3/2} \qquad (1)$$

can be rewritten with all the constant terms separated, as:

$$M_J = \frac{9}{4(2\pi)^{1/2}} \left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{n^{1/2} m^2}$$

Question 3

The value of A is

$$\left(\frac{9}{4\times(2\pi)^{1/2}}\right)\times\left(\frac{1.38\times10^{-23}}{6.67\times10^{-11}}\right)^{3/2} = 8.45\times10^{-20}$$

The units of A are obtained from the units of k and G (all other terms have no units).

The units are

$$\left(\frac{\mathrm{J\,K^{-1}}}{\mathrm{N\,m^2\,kg^{-2}}}\right)^{3/2}$$

which can be written

$$\left(\frac{\mathrm{N\,m\,K^{-1}}}{\mathrm{N\,m^2\,kg^{-2}}}\right)^{3/2} = (\mathrm{m^{-1}\,K^{-1}\,kg^2})^{3/2}$$

$$= \mathrm{m^{-3/2}\,K^{-3/2}\,kg^3}$$

Question 4

One way to write the formula for A is

```
=(9*(1.38E-23/6.67E-11)^1.5)/(4*(2*PI())^0.5)
```

There are many alternatives, but they should all give the correct answer!

A simpler way is to use the cell references for the values of k, G and π, which would be written

```
=(9*(C8/C9)^1.5)/(4*(2*C10)^0.5)
```

for the spreadsheet in Figure 2.

Question 5

The formula for the Jeans mass in the spreadsheet is

```
=C11*C17^1.5/(C15^0.5*C16^2)
```

Question 6

(a) Entering $m = 1m_\mathrm{H}$ ($= 1.67\times10^{-27}$ kg) for neutral hydrogen, $T = 30$ K and $n = 10^{10}\ \mathrm{m^{-3}}$ gives a Jeans mass of $7.9M_\odot$. The cloud mass is smaller than the Jeans mass so it will not collapse.

(b) Entering $m = 2m_\mathrm{H}$ ($= 3.34\times10^{-27}$ kg) for molecular hydrogen, $T = 30$ K and $n = 10^{10}\ \mathrm{m^{-3}}$ gives a Jeans mass of $2.0M_\odot$. The cloud mass now exceeds the Jeans mass so it is likely to collapse.

Question 7

Rearranging the form of the Jeans mass equation from Question 2:

$$M_\mathrm{J} = \frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2}\times\frac{T^{3/2}}{n^{1/2}m^2}$$

gives $n^{1/2} = \frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{M_J m^2}$

so $n = \left[\frac{9}{4(2\pi)^{1/2}}\left(\frac{k}{G}\right)^{3/2} \times \frac{T^{3/2}}{M_J m^2}\right]^2 = \left[A \times \frac{T^{3/2}}{M_J m^2}\right]^2$

The formula will therefore be

```
=(C11*C17^1.5/(C15*C16^2))^2
```

Question 8

(a) The critical density for collapse is 1.95×10^9 m^{-3}. If the cloud is compressed to a density greater than this value it will collapse.

(b) The critical density for collapse is 1.22×10^8 m^{-3}.

The critical density does not need to be so high for a molecular hydrogen cloud. The gravitational forces, which must overcome the outward pressure in the gas for collapse to occur, depend on the total mass of the cloud, the masses of individual particles and their separations (as defined by the number density). The total mass of each cloud is the same, but the mass of each particle in the molecular hydrogen cloud is higher than in the neutral hydrogen cloud. A particle at the edge of the molecular hydrogen cloud does not therefore need to be so close to the centre of mass of the cloud to feel the same gravitational force as a particle at the edge of the neutral hydrogen cloud. The molecular hydrogen cloud can therefore be larger, i.e. have a lower number density than the neutral hydrogen cloud, for collapse to occur.

Question 9

(a) The critical temperature below which the cloud is likely to collapse is 40 K.

(b) The critical temperature is 1.8 K.

(c) The critical temperature is 9 K.

The critical temperature for the low density diffuse cloud in (b) is lower than for the higher density cloud in (a) with similar properties. The two clouds have the same total mass and the same particle mass, but the mean separations of the particles in the diffuse cloud are greater than in the dense cloud. This means that the diffuse cloud in (b) must be larger than the dense cloud in (a). A particle at the edge of the diffuse cloud will therefore be further from the centre of mass of the rest of the cloud and the gravitational force on it will be lower. In order for collapse to occur the gas pressure (as determined by the motions of the particles, and hence the temperature) must therefore also be lower.

If the mass of the diffuse cloud is raised but the number density remains unchanged then the cloud size must also increase. The gravitational force on a particle at the edge of the cloud increases because the effect of increase in mass more than compensates for the increase in distance from the centre of mass. (The gravitational force on the particle is proportional to M/R^2. Since R is proportional to $M^{1/3}$, then the gravitational force is proportional to $M/(M^{1/3})^2$ or $M^{1/3}$. So, as mass increases the gravitational force increases.) As the cloud mass is increased the gravitational forces are therefore increased and the temperature does not need to be so low for collapse to occur.

(Note: this analysis is very simplistic and ignores all the other factors which may govern the collapse of interstellar clouds.)

S282 Astronomy

A delicate balancing act

Study time: 20 minutes

Summary

In this activity you will view a video sequence that describes hydrostatic equilibrium; how pressure gradients support stars against gravity. The first part of the sequence discusses the forces due to gas pressure and radiation pressure that provide the pressure gradient to support 'normal' stars. In the second part of the sequence degenerate matter, which supports highly evolved stars, is explained.

The first part of this sequence (up to 4:35) is best watched after you have read to the end of Chapter 6 of *An Introduction to the Sun and Stars*. The second part (4:35 onwards) should be watched after you have been briefly introduced to degenerate matter in Section 7.2.4 of the book. The evolved stars in which it dominates are described in detail in Chapter 9. You may wish to watch this sequence again when you have completed Chapter 9.

Learning outcomes

- Understand the concept of hydrostatic equilibrium as a balance between the force of gravity and a pressure gradient.
- Recognize that in 'normal' stars this pressure gradient may be maintained by gas pressure or radiation pressure.
- Appreciate that the pressure gradient which supports some stars is provided by degeneracy pressure.

The activity

- Open the S282 Multimedia guide and then click on A delicate balancing act under the 'Stars' folder in the left-hand panel.
- Press the Start button to start the video sequence.
- After viewing the first segment (up to time 4:35) the screen will go blank. Pause the player and answer the following question.

Question 1

Try to summarize the argument so far in a few sentences.

You can now watch the second segment or wait until you have read to the end of Section 7.2 of *An Introduction to the Sun and Stars*. In this segment the concept of degeneracy pressure is introduced.

Notes

The video sequence can be summarized as follows (the timings are those that appear on screen as the topics begin).

00:00 Demonstrations and animations illustrate how pressure gradients support stars against gravity and how pressure (due to particles' motion) and temperature gradients are interrelated.

03:50 Radiation pressure is introduced.

04:35 The pause in the tape for Question 1.

04:50 Degeneracy pressure is introduced. Here is a summary of the main points regarding degeneracy pressure:

- Degeneracy pressure is commonly found in some of the late stages of stellar evolution – in white dwarfs and neutron stars.
- Degeneracy pressure is a quantum phenomenon; it is not noticeable in everyday life, but is very important in some extreme physical conditions.
- Quantum effects prevent two (or more) identical particles having the same energy. However, two particles of the same type but with opposite spins may have the same energy. (An analogy of two particles of the same type but opposite spin is a pair of gloves – the gloves are identical except that one is left-handed and the other right-handed.)
- This quantum restriction was demonstrated in the sequence by balls of two different colours. At most, one ball of each colour could occupy the same pigeon hole. This meant some balls were forced up into the higher levels.
- In white dwarfs and neutron stars, degeneracy pressure predominates. As a consequence, pressure is not proportional to temperature: it is far *less* sensitive to temperature than in a non-degenerate star.

07:30 Sequence ends.

Video credits

Presenter – Jocelyn Bell Burnell (The Open University)

Producer – Cameron Balbirnie (BBC)

Course Team consultant – Alan Cooper

Answer to Question 1

Your synopsis should contain the following points:

- Pressure difference across a small element of a star opposes the gravitational force on the material in that element (more precisely, pressure force = pressure difference × area of element).
- The pressure may be due to collisions of gas particles with gas particles or may be due to photons colliding with gas particles. Both types of process occur in all normal stars, but particle collisions dominate if the star's mass is under $5M_{\odot}$ and photon collisions dominate above this.

  In a normal star, gas pressure is proportional to temperature (more completely, pressure is proportional to density × temperature; see *An Introduction to the Sun and Stars*, Equations 6.1 and 6.12). Radiation (photon) pressure is even more sensitive to temperature (Equation 6.11).

S282 Astronomy

Variable stars

Study time: 2 hours

Summary

In this spreadsheet activity you will analyse some results of observations of Cepheid variable stars. You will first investigate the Cepheid period–luminosity relation using data from stars in the Small Magellanic Cloud. (*Note there is an error in the summary on the multimedia guide page, which incorrectly says 'Large Magellanic Cloud'*). Then you will determine the period of an unknown star by plotting a light curve of a month's observations. This period will be used to determine the luminosity and hence an estimate of the distance to the star.

It is recommended that you attempt this activity after you have read to the end of Chapter 3 of *An Introduction to the Sun and Stars* and after you have completed the spreadsheet activities 'Sunspot number' and 'Stellar distance and motion'.

Learning outcomes

- Develop your understanding of how Cepheid variable stars are used to determine distances.
- Develop your spreadsheet skills, particularly with the use of formulae and graph plotting and interpretation.
- Use a spreadsheet to determine the period of a variable star when the light curve is randomly sampled.

Background to the activity

Cepheids are pulsating variable stars. They exhibit regular changes in luminosity, temperature and radius due to instabilities in the outer layers of the stars. They are named after the first to be discovered, δ Cephei, in 1784, but it would be over a century before their importance as standard candles, (objects with known luminosity that can be used to determine astronomical distances) was recognized.

Henrietta Leavitt (Figure 3.32 in *An Introduction to the Sun and Stars*) examined hundreds of photographic plates obtained between 1893 and 1906 at Harvard College's observatory in Peru to produce a catalogue of 1777 variable stars in the Magellanic Clouds (two regions that look like detached parts of the Milky Way, visible in the southern hemisphere that are now known to be small external galaxies). A total of 16 stars appeared in enough plates for their periods to be determined, and at this stage she noticed that the brighter ones appeared to have the longest period.

By 1912 she had obtained periods and magnitudes for 25 stars in the Small Magellanic Cloud (SMC) which confirmed this period–luminosity relation. She recognized the characteristic shape of the light curves and wrote:

> Since the variables are probably at nearly the same distance from the Earth, their periods are apparently associated with their actual emission of light, as determined by their mass, density and surface brightness … It is to be hoped, also, that the parallaxes of some variables of this type may be measured.

This only hinted at the great importance that Cepheids would attain in the determination of distances to external galaxies.

In the first part of this exercise you will use data for the 25 stars measured by Leavitt to plot the apparent magnitude *m* versus log (period) and obtain a best fit straight line of the form

$$m = c + b \log P \tag{1}$$

where *b* is the slope of the line and *c* is the intercept. This graph, plotted by Leavitt, led to her confirmation of the period–luminosity relation for the SMC variables. Without any independent measurement of the distance of the SMC it was not possible at the time to calibrate this relationship in terms of the intrinsic brightness of the stars. Since the SMC stars are all at the same distance then we know from Equation 3.16 in *An Introduction to the Sun and Stars* that

$$M = m - 5 \log d + 5 \tag{2}$$

(where *d* is in parsecs) tells us that $M = m$ + a constant and so

$$M = a + b \log P \tag{3}$$

where the slope, *b*, is the same as in Equation 1 and the new constant, *a*, provides the calibration of the period–luminosity relation.

Danish astronomer Ejnar Hertzsprung (Figure 4.2a in *An Introduction to the Sun and Stars*) realized that if the period–luminosity relation could be calibrated, (i.e. the value of the constant *a* determined) then the absolute magnitudes, *M*, of Cepheids could be determined directly from their periods. Comparison with their measured apparent magnitudes, *m*, would then yield their distances, *d*, using Equation 2.

The actual determination of the zero point of the calibration was a difficult process as no Cepheids were close enough for parallax to be determined. Even today there is some uncertainty in the value of *a*.

This activity is divided into two parts. If you don't want to complete the activity in one go then you can save your data and return at any point. However it is recommended that you complete each part in one sitting if possible so you don't lose track of what you are doing.

You will be using data provided in an existing spreadsheet. You should now be familiar with the use of spreadsheets. For a reminder of the basic functions or for more information, refer to the *Using Spreadsheets* guide on the course website.

The instructions given here assume that you will be using the StarOffice™ package that is supplied on the OU Online Applications CD-ROM. If you are already familiar with using another spreadsheet package (such as Microsoft Excel) you may want to use that to carry out the activity (we have also supplied the required data file in Excel format). However, before starting, you should be aware that these notes only give instructions on how to manipulate the StarOffice spreadsheet.

Part 1 Cepheids in the Small Magellanic Cloud

Open the raw data file

Before you start it would be a good idea to set up a folder in which you can store the results of your work.

The raw data for this activity is contained in a file called 'S282 Variable star.sxc'. (The Excel version of the file is called 'S282 Variable star.xls').

- Start the S282 Multimedia guide program, open the folder called 'Stars' then click on the icon for this activity ('Variable stars').
- Press the Start button to access the folder on the DVD containing StarOffice and Excel versions of the file containing the raw data.
- Open the file you wish to use by double-clicking on it.

Save a copy of the file

Before you can make changes to the file, you must save it to your hard disk.

- Use the File | Save as... menu command to save a copy of the spreadsheet into your work folder.

As you make changes to the spreadsheet you should save your work regularly to prevent any changes from being lost. From time to time make a backup copy of your work (using a different filename) in case you need to go back to an earlier stage.

Preparing the data

The spreadsheet contains two worksheets, labelled Cepheid light curve and PL relation on the tabs at the bottom of the spreadsheet.

- Select the PL relation worksheet by clicking on the tab. You should see the table shown in Figure 1 (*overleaf*).

The table contains apparent magnitudes for the maximum and minimum brightness and the pulsation period in days for the 25 Cepheids identified by Henrietta Leavitt in the Small Magellanic Cloud. (*You should change the label 'LMC' to 'SMC' here.*) In order to determine the slope of the period–luminosity relation for these stars you will need to plot the mean apparent magnitude against the logarithm of the period. The column headings are already prepared for this.

	A	B	C	D	E
1	S282 Activity – Variable Stars				
2					
3	Your name		Date	today's date	
4					
5	Data for the 25 stars in the LMC measured by Henrietta Leavitt				
6					
7	Period /days	Magnitude at maximum	Magnitude at minimum	Log (Period /days)	Mean magnitude
8	1.26	14.81	16.09		
9	1.66	14.81	16.09		
10	1.76	14.81	16.39		
11	1.87	15.1	16.29		
12	2.1	14.8	16		
13	2.21	14.69	15.57		
14	2.94	14.4	15.69		
15	3.5	14.69	15.91		
16	4.37	14.58	16.11		
17	4.62	14.3	15.3		
18	5.12	14.31	15.5		

Figure 1 The PL relation worksheet prior to carrying out any calculations. (Note there is a typographical error in the spreadsheet – 'LMC' should be 'SMC'.)

- In cell D8 type the formula `=log10(A8)` and press Enter or click the symbol to determine the logarithm (to the base 10 – see Box 3.2 of *An Introduction to the Sun and Stars*) of the period in column A8. You can then apply this formula to each cell in the column by *dragging* (as described below).
- Select cell D8, which contains the formula, click on the little black square in the bottom right-hand corner of the cell and drag down to row 33, keeping the mouse button held down.
- This will replicate the formula all the way down the column, updating the row numbers in the formula. You should now have a column containing log(period) values for all the stars.

(At this point you may want to reformat the column to show more decimal places – refer to *Using Spreadsheets* if you cannot remember how to do this.)

Column E is labelled Mean magnitude.

Question 1

What formula do you need to type in cell E8 to calculate the mean apparent magnitude?

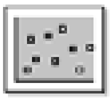
- Apply this formula to the remaining cells in the column as before.

Create the chart

The data in columns D and E can now be used to plot a chart of magnitude versus log(period). If you have completed the 'Sunspot number' activity you will already be familiar with plotting graphs in a spreadsheet. The instructions are repeated here:

- Highlight both columns by clicking on cell D8 and dragging all the way down and across to cell E33.
- From the main menu, select Insert | Chart ....

- A box labelled AutoFormat Chart will appear. You don't need to change anything here, so select Next>> to move onto the next step.
- From the Choose a chart type options, select XY Chart, i.e.



- followed by Next>> to move onto the next step.
- From the Choose a variant options, select the same icon (Symbols only). If you prefer to have gridlines on both the X and Y axes, or neither, you can select the appropriate boxes. Select Next>> to move onto the next step. (Note that while going through these steps, you can always press Back to go to an earlier step if you need to correct something.)
- In AutoFormat Chart make the following changes:

  The tick-box next to Legend should be cleared (i.e. without a tick).

  Make sure the X Axis and Y Axis tick-boxes are ticked – this will cause the axis titles to be displayed.

  Insert the text for titles as follows:

Chart title:	SMC Cepheids (*Note*: LMC should be SMC)
Axis title / X Axis:	Log(Period/days)
Axis title / Y Axis:	Apparent magnitude

- Finally, press the Create button – this will draw the chart.

Formatting the chart

Having created the chart you will need to format it to improve the way in which it displays these data, and maybe tidy it up to make it visually more appealing. In order to make changes, the chart must be *selected*: there are two different levels of selection, signified by different borders around the chart:

- To resize, move or delete the chart: single click; the chart has green selection handles.
- To edit or format the chart: double-click; the chart has grey border and small black squares.
- To deselect the chart: click on any other cell in the spreadsheet.

Note that to swap between the 'resize, move, delete' selection and the 'edit, format' selection it is necessary to deselect the chart, before selecting it again.

The first thing that you are likely to want to do is to resize the chart to make it larger. To do this select it by single clicking and then dragging one of the green selection handles as required.

To change other aspects of the appearance of the chart you will need to make sure that the chart is selected for formatting (grey border and black squares). You can change the formatting of just about any element of the graph by double-clicking on the part you want to change. As you move the mouse pointer over the chart a small label will appear telling you which item will be modified (you may need to be quite precise with the mouse to select the item you want!). In this way you can change colours, add or remove background shading, gridlines and so on.

There are many different things that you can do to change the appearance of your chart. Some of them will be cosmetic, i.e. to make the chart more visually appealing, or easier to use. There is, however, one aspect of this chart that does

not satisfy the normal astronomical convention for plotting magnitudes. The magnitude scale has the brightest objects with the most negative magnitudes. Spreadsheet plotting routines automatically show the axes with the largest numerical value at the top. Although some spreadsheets (such as Microsoft Excel) allow the axes to be reversed, this facility is not present in StarOffice. It is not important for this part of the activity, but you should remember that the brightest stars appear at the *bottom* of the figure.

If you want to re-apply any of the chart type selections or titles choose Format | AutoFormat... from the main menu; this will take you through the AutoFormat Chart steps again.

Changes you may want to make are removing the grey background (double-click on chart area) adjusting the axes (click near the axes or use Format | Axis | X Axis or Format | Axis | Y Axis) and adding a border around the chart and plotting area (use Format | Chart area and Format | Chart wall respectively). Experiment with the various formatting options until you are happy with the appearance of your graph.

Question 2

Comment on the relationship between log(period) and apparent magnitude for the 25 stars plotted on your graph.

The period–luminosity relation

You can now use your graph to determine how well the data is represented by Equation 1:

$$m = c + b \log P$$

Question 3

What do the constants *b* and *c* represent on your graph?

The spreadsheet contains software that automatically determines the best fit straight line to a set of data as follows:

- Ensure the chart is selected then select Insert | Statistics from the main menu. (Alternatively you can double-click on the data points to reveal the Data Series screen and select the Statistics tab.) The box shown in Figure 2 (*overleaf*) will appear.
- In the Regression Curves box select the Linear regression type fit from the different types of line fitting available.
- Click on OK and the best fit line will appear.

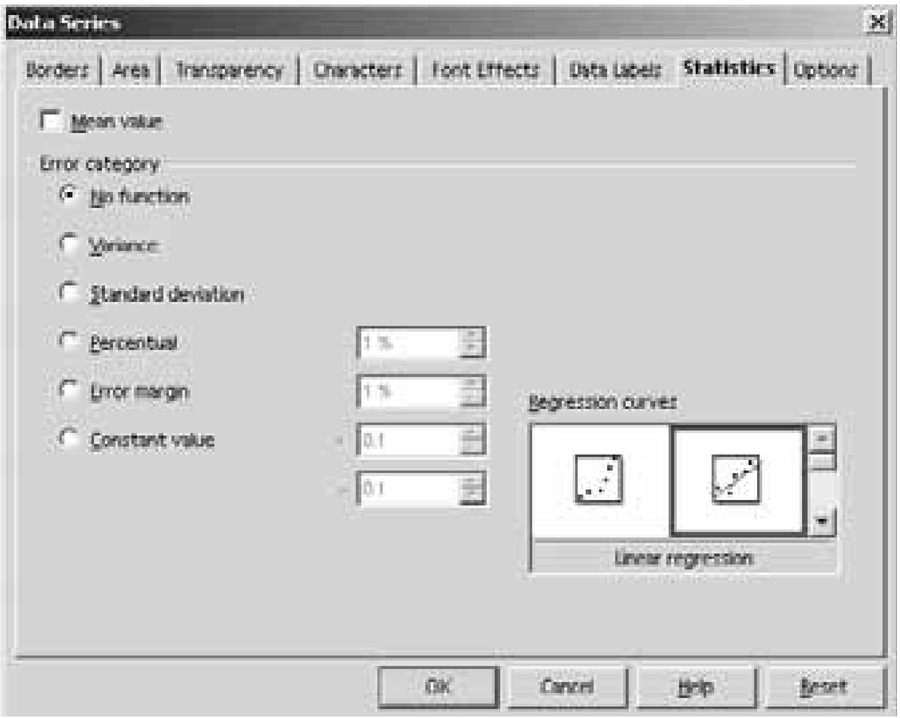


Figure 2 Window for adding straight line fit to data on a graph.

Question 4

The magnitudes of the stars do not perfectly fit the straight line. Assuming the period–luminosity relation can be represented perfectly by a straight line on this type of plot can you explain why you would not expect the stars to fit it exactly?

Question 5

The distance of the SMC is now known to be approximately 55 kpc with a diameter of about 3 kpc. Use Equation 2 to show that this is compatible with the data and your answer to Question 4.

In fact, the period–luminosity relation plotted for many Cepheids will show scatter for a number of other reasons including:

(i) The absorption of light by interstellar dust (see Section 4.3.3 in *An Introduction to the Sun and Stars*) will affect the observed apparent magnitudes and must be corrected for.

(ii) The structure of a star depends primarily on its mass, but also its composition and energy source (Chapter 6 of *An Introduction to the Sun and Stars*). Although Cepheids are all at a similar stage of their evolution, with the same nuclear energy source (as you will see in Chapter 7 of *An Introduction to the Sun and Stars*) they may have different *initial* compositions and therefore slightly different structures and pulsation periods for a given mass (this effect is described in Chapter 9 of *An Introduction to the Sun and Stars*). This effect is sufficiently important that two classes of Cepheids exist with two different period–luminosity relations. We will not consider this further in this activity. If no distinction is made then we assume that the Cepheids formed in regions similar to that in which the Sun formed (and should correctly be called *classical Cepheids* or *Type 1 Cepheids*).

The 25 stars you have examined in the SMC do not provide a sufficiently large number of stars to represent the whole of the Cepheid population, and hence do not allow an accurate determination of the period–luminosity relation. They were however, as you have seen, sufficient for Henrietta Leavitt to recognize that the relationship existed. The exact details of the period–luminosity relation are being continuously refined as improved observational data become available. For example the Hipparcos mission (see Section 3.2.2 of *An Introduction to the Sun and Stars*) provided parallaxes for stars at greater distances and significant strides were made in determining astronomical distances.

For the next part of the activity we will use a recent evaluation of the period–luminosity relation:

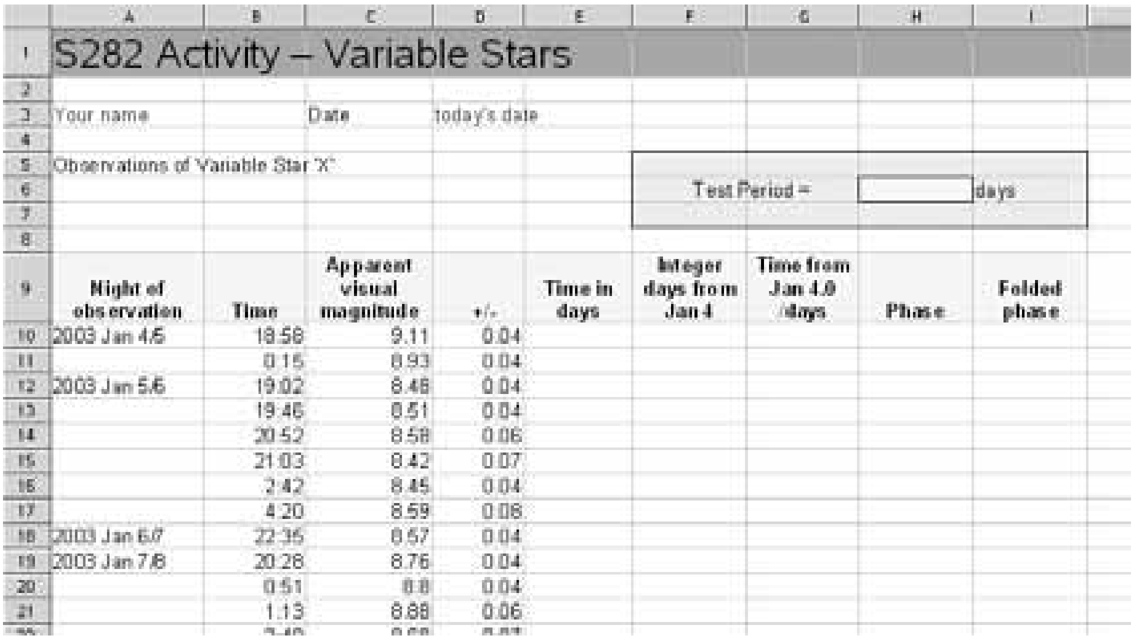
$$M_V = -2.8 \log P - (1.4 \pm 0.2) \qquad (4)$$

where P is the period in days.

Part 2 The light curve and distance of an unknown Cepheid

In this part of the activity you will analyse some observed data for an unknown Cepheid to determine its pulsation period and hence estimate its absolute magnitude and distance.

- Open the second worksheet in the spreadsheet, which is labelled 'Cepheid light curve'. You should see the table shown in Figure 3:



S282 Activity – Variable Stars

Your name | Date | today's date

Observations of Variable Star 'X'

Test Period = [ ] days

Night of observation	Time	Apparent visual magnitude	+/-	Time in days	Integer days from Jan 4	Time from Jan 4.0 /days	Phase	Folded phase
2003 Jan 4/5	18:58	9.11	0.04					
	0:15	8.93	0.04					
2003 Jan 5/6	19:02	8.46	0.04					
	19:46	8.51	0.04					
	20:52	8.58	0.06					
	21:03	8.42	0.07					
	2:42	8.45	0.04					
	4:20	8.59	0.08					
2003 Jan 6/7	22:36	8.57	0.04					
2003 Jan 7/8	20:28	8.76	0.04					
	0:51	8.8	0.04					
	1:13	8.88	0.06					

Figure 3 The Cepheid light curve worksheet prior to carrying out any calculations.

The star has been observed for a period of a month. The date, time, magnitude and estimated uncertainty in the magnitude are provided.

Folded light curves

The period of pulsation and therefore change in brightness is unknown but if the star is a Cepheid it will be in the range of 1 to 100 days. Even if the period is very short then up to half the light curve will be visible in one night. If the period is as long as 100 days then the magnitude will be virtually constant over the course of

one night but will gradually change from night to night. If the period is a few days then the magnitude will appear to change randomly from night to night. Although it will be possible to distinguish between these cases it is highly unlikely that you will be able to determine the period simply by looking at the light curve.

When magnitudes are measured for many small parts of a light curve, which may span a number of pulsation cycles it is convenient to plot what is called a *folded* light curve. Instead of a plot of time in hours or days versus magnitude, the time, t, is divided by the period, P, to produce the *phase* (t/P). The phase therefore consists of two parts: the whole number, which tells you the number of periods that have passed since time $t = 0$, and the decimal part, which tells you what part of the cycle you have reached. The light curve can then be plotted as the decimal part of the phase versus magnitude. This has the advantage of overlaying all the data for many pulsations into one period so that details in the structure of the light curve can be seen.

Of course, you do not initially know the period, but this light curve can be used to determine it as you will see.

First, however, you need to convert each date and time into a phase.

Preparing the data

Columns A and B contain the date and time of each observation. In order to perform calculations these need to be converted into a single number. It is convenient to choose a fixed date and time as zero and to then calculate all other times in units of hours or days from this point. The start of the first day of observation is the most sensible to use so 4 Jan 2003 at 0:00 can be set as $t = 0$ and time measured in days from this point.

Column B contains the time in the format hh:mm which is recognized by StarOffice as a time. If it is used in a calculation it will automatically be converted into decimal *days*. Unfortunately the date in column A is not written in a convenient form that the spreadsheet recognizes as a date so we will need to determine the time t in stages. (You can see which formats are recognized by looking at the options in Format | Cells and selecting date.)

The value for cell E10, which is the time (cell B10) in days, is therefore the trivial formula `=B10`. Apply this to the whole column.

Question 6

How many decimal places are appropriate for the data in column E?

Adjust the formatting of the column so that the data appear as numbers with the appropriate number of decimal places.

Column F is the number of whole days from 4 Jan 2003. Enter these values manually by inspection of column A. You need to take great care here because:

(i) the day number changes at midnight so that cell F10 contains the value 0, F11 to F15 contain 1, etc.)

(ii) there are some days when no data have been taken (presumably because of bad weather) and there are therefore gaps in the day number.

As a check that you have performed this correctly, cell F55 should contain the value 12 and cell F76 should contain the value 23.

Column G contains the total time in days from Jan 4.0.

Question 7

What is the formula to be written in cell G10 and what is the value?

- Apply this to the whole column by dragging and reformatting to show the appropriate number of decimal places.

Column H is where you will calculate the phase. This is calculated using the (currently unknown) period. The yellow box above the table contains the value of the period in cell H6 (Figure 4) in days. Enter an initial guess (say 1) here.

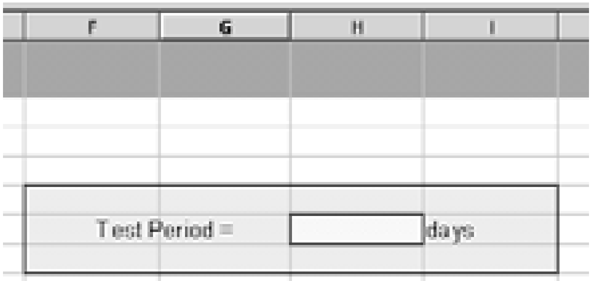


Figure 4 Location in the spreadsheet where the pulsation period is entered.

- Enter the formula for the phase, `=G10/$H$6`, in cell H10.

The $ symbols before the 'H' and the '6' in the cell address for the period signify that they will not be updated when the formula is dragged. This is called an absolute cell reference. (Note if the cell shows `Err.503` then an attempt to divide by zero has been made, i.e. you have not entered a value in the period box, cell H6.)

- Apply this formula to the whole column by dragging.
- Inspect the entry in cell H11. You can see it should read `=G11/$H$6` as required. If the $ symbols had not been entered it would read `=G11/H7`, which would give an erroneous answer.
- Reformat the column to the appropriate number of decimal places.

You now have values of phase for each date and time. However, this is not sufficient to plot a folded light curve. You need to remove the integer (whole number) part of the phase so that all periods are plotted on top of each other. This is achieved by entering the formula `=H10-INT(H10)` in cell I10. Apply this to the whole column by dragging and reformatting to the appropriate number of decimal places.

You now have all the data ready to plot the folded light curve.

Plotting the folded light curve

The procedure for plotting as described above for the magnitude versus period graph can be repeated with a few changes. Before plotting the light curve there are three factors to consider.

First, you will want to plot the magnitude on the vertical axis (Y axis) and folded phase on the horizontal axis (X axis). The procedure used in Part 1 of this activity requires data in adjacent columns with the X axis data first. It is possible to select non-adjacent columns and correct for data being in the wrong order, but a simpler solution is to copy the magnitude column next to the folded phase column. Before you do this you need to consider the following point.

Second, because of the limitations of StarOffice, the light curve will appear upside down. This is now important as you will want to identify the characteristic pattern for a Cepheid light curve and have the brightest magnitudes at the top. It therefore makes sense to multiply the magnitudes by –1 to reverse the scale and remember to ignore the negative sign in the plot:

- Enter the formula `=-C10` in cell J10 and apply to the column by dragging. Adjust the formatting and label the column '–Magnitude for plotting'.

Third, the folded light curve will show data from phase 0.0 to 1.0. This means that there will be one maximum and one minimum visible on the light curve. In order to see the pattern of the light curve more easily and judge when the shape is most consistent it is convenient to plot two periods (i.e. repeat the data between phase 1.0 and 2.0. This means that wherever the maximum or minimum lies there will be one complete cycle visible.

- Select cell I78 and enter the formula `=I10+1`. Apply this formula to the entire data set by dragging down to cell I144. The cells I78 to I144 will now contain the folded phase values between 1 and 2. Reformat if required.
- Next you will need to copy the magnitude data which apply to these folded phases. Select cell J78 and enter the formula `=J10`. Apply this formula to the entire data set by dragging down to cell J144.

The data in columns I and J can now be used to plot a chart of magnitude versus folded phase.

- Highlight both columns by clicking on cell I10 and dragging all the way down and across to cell J144.
- From the main menu select Insert | Chart ...
- Select Next>> in the AutoFormat Chart.
- From the Choose a chart type options select XY Chart, followed by Next>>.
- From the Choose a variant options select Symbols Only, followed by Next>>.
- In AutoFormat Chart clear the tick-box next to Legend, and make sure the X Axis and Y Axis tick-boxes are ticked.

  Insert the titles:

Chart title:	Light curve for star X
Axis title / X Axis:	Phase
Axis title / Y Axis:	–Apparent visual magnitude

- Finally, press the Create button to draw the chart. (You will probably see an apparent error message stating that the X-axis values must be sorted. It is not necessary so click No.) The chart will now appear near the top of your worksheet (you will have to scroll back to see it).
- Reformat the chart as desired.

At this stage the data will appear as a scatter (Figure 5) since the correct period has not yet been found.

There is one further factor that should make it easier to judge the best fit period. Each observed magnitude has an associated uncertainty listed in column D. It is best to add these uncertainties to the plotted light curve as error bars.

Unfortunately the StarOffice spreadsheet does not allow *different* error bars to be plotted for each point so it is necessary to plot a *typical* value for each point.

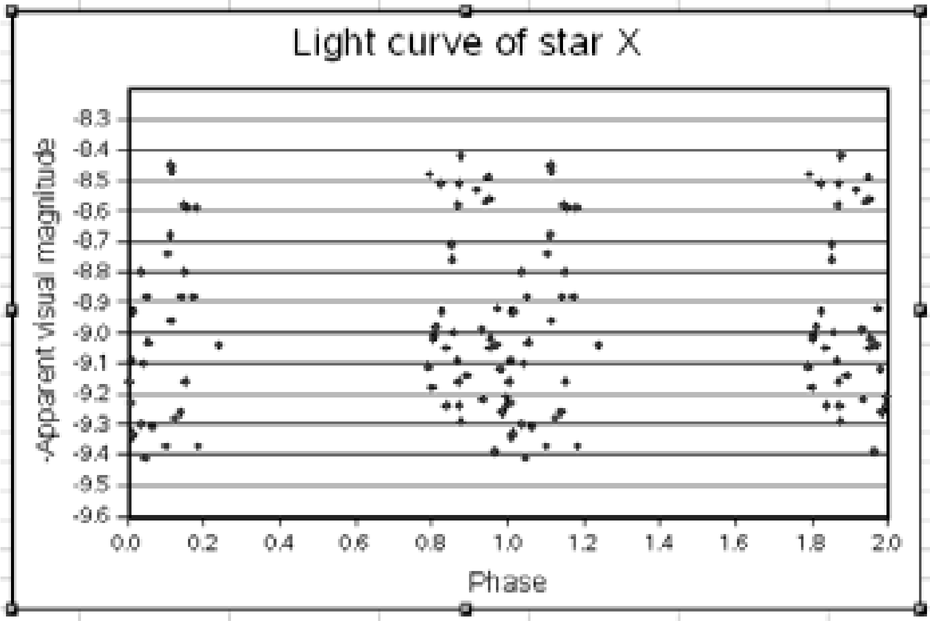


Figure 5 Light curve after initial plotting and formatting.

Question 8

Using the spreadsheet data, what is a representative value for the typical uncertainty in an observed apparent visual magnitude? Can you think of a reason why all the values which differ from this one are larger?

In StarOffice error bars are usually added using Insert | Statistics from the main menu. (Alternatively you can double click on the data points to reveal the Data Series screen and select the Statistics tab) as shown in Figure 2. If Constant value is selected from the Error category choices then the uncertainties can be entered in the + and – boxes. However, if you try this you will discover that StarOffice will not allow you to enter values smaller than 0.1. There is another way around this problem that involves changing the symbols used in the graph plotting, as follows:

- Double-click to select the graph for editing.
- Place the cursor over one of the data points on the graph and double click. A Data Series window will open.
- Select the Borders tab and the screen shown in Figure 6 should appear.

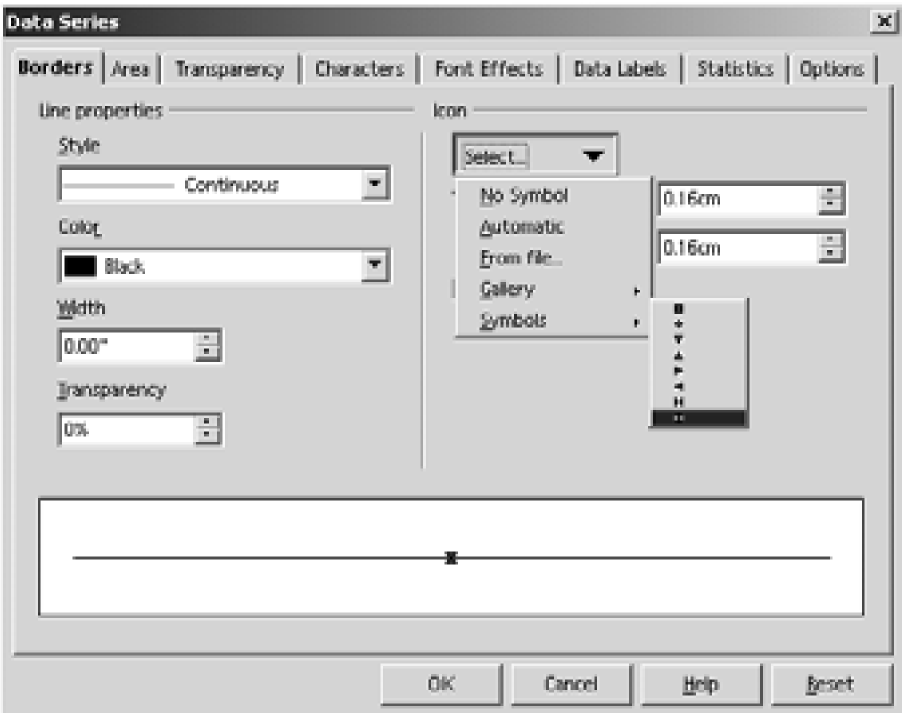


Figure 6 Window for formatting the data symbols plotted on a graph.

- You can adjust the symbol shape using the Select button and taking the Symbols option. Choose the bottom symbol (which you will see resembles an error bar) to replace the default diamond shaped symbol (as shown in Figure 6).
- De-select the Keep ratio box and adjust the symbol size using the Width and Height boxes; 0.1 cm is a reasonable value for the Width. Try 0.4 cm as a first attempt at the Height.
- Click the OK box to view the result. You will need to experiment with the Height to obtain the best value to match plus and minus the uncertainty from Question 8 on your particular plot. (This is rather fiddly!)

The light curve should now look like Figure 7.

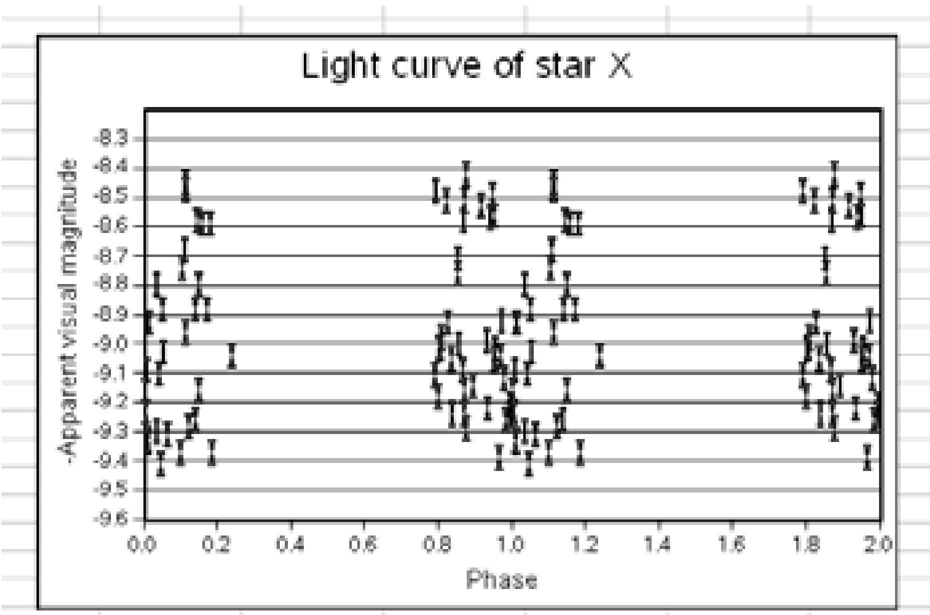


Figure 7 The folded light curve before searching for the best period.

Determining the period of star X

You can now experiment with entering new values for the period in cell H6. Examine the appearance of the folded light curve until you think you have the best representation of a smooth light curve. You will need to confirm that it really is a Cepheid variable (it will not surprise you to learn that it is!) based on the light curve shape (see Figure 3.31 in *An Introduction to the Sun and Stars* if you need reminding).

Note that the uncertainties listed in column D represent the estimated uncertainties in the data so they give an indication of how much scatter you might expect. The points with the larger uncertainties may well lie further from the general trend of the light curve. When examining the light curve, you can check if points that do not fit the trend well correspond to these less reliable data. A more sophisticated spreadsheet package would allow the actual uncertainties to be plotted making this process easier.

Question 9

What is your estimate of the pulsation period of the Cepheid, star X? What is your estimate of the uncertainty in this value?

Estimating the distance of the Cepheid

Using the period–absolute magnitude relation as given in Equation 4:

$$M_V = -2.8 \log P - (1.4 \pm 0.2)$$

you can derive the absolute visual magnitude of star X. The uncertainty in the absolute visual magnitude will be a combination of the uncertainty in the relation itself (0.2) and that resulting from the uncertainty in the period.

Question 10

What is your derived value of the absolute visual magnitude of star X and your calculated uncertainty in this value?

(Hint: derive the maximum and minimum likely values of M_V using $P \pm \Delta P$ in Equation 4 without the ± 0.2. Then combine the resultant uncertainty with the ± 0.2. See previous spreadsheet activities if you are still unsure how to do this.)

Question 11

What is your estimate of the mean apparent visual magnitude of star X from your light curve? What is your estimated uncertainty in this value?

Question 12

Use Equation 2 to determine the distance of star X and the uncertainty in this value. (Hint: to derive the uncertainty in d, derive the uncertainty in $(m_V - M_V)$ and then determine the value of d for the maximum and minimum values.)

Question 13

What assumption has been made in deriving this distance?

Answers to questions

Question 1

The formula for mean magnitude should be `=(B8+C8)/2`.

Question 2

The data do show a trend with the longest period stars having the brightest magnitudes. It is approximately a straight line but there is some scatter in the data.

Question 3

The constant *b* is the gradient of the best fit line on your graph. The constant *c* is the intercept with the Y axis, i.e. the apparent magnitude for a value of log(period/days) = 0, which corresponds to the apparent magnitude for a star with a period $P = 1$ day.

Question 4

Your graph should look similar to Figure 8.

There are two reasons why the stars would not be expected to lie exactly on the best fit line:

1. There will be observational errors present in the data (although we have not been supplied with an estimate of the uncertainties in the data to check whether they are consistent with the scatter).
2. Although we have assumed all the stars are at the same distance (so that the apparent magnitudes differ from the absolute magnitudes by exactly the same amount), the SMC does have a finite size. Any star might lie at the front or the back of the SMC as seen from the Earth and therefore lie slightly closer (or further away) than the average. This will make its apparent magnitude slightly smaller (or larger) than it would if it lay at the average distance.

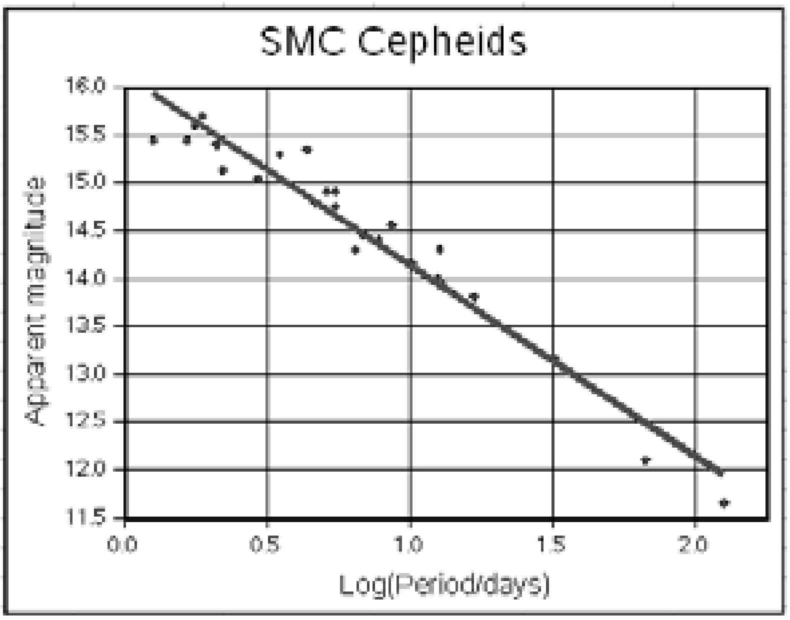


Figure 8 The relationship between apparent magnitude and log(period) for the SMC Cepheids observed by Henrietta Leavitt.

Question 5

If the diameter of the SMC is not negligible compared with the distance from the cloud to the Earth then we can calculate the effect this would have on the apparent magnitude of a star.

Using Equation 2:

$$M = m - 5 \log d + 5$$

The absolute magnitude M of a star is a measure of its luminosity. We can work out the effect on the apparent magnitude m, of changing d. Rearranging Equation 2 gives

$$m = M + 5 \log d - 5$$

As the cloud has a diameter of 3 kpc, a star at the front of the cloud will have a distance of 1500 pc *less than* the mean distance of the stars in the cloud and a star at the back of the cloud will have a distance 1500 pc *more than* the mean distance of the stars in the cloud

so $m_{\text{min}} = M + 5 \log(55\,000 - 1500) - 5 = M + 18.64$

and $m_{\text{max}} = M + 5 \log(55\,000 + 1500) - 5 = M + 18.76$

The finite depth of the SMC can therefore introduce a scatter of up to 0.12 magnitudes in the observed apparent visual magnitudes compared to the best fit line.

Your graph shows that this can explain only part of the scatter in the data.

Question 6

The times in column B are quoted to the nearest minute and can therefore be no more accurate than ±0.5 minutes. 0.5 minute = $1/(2 \times 24 \times 60) = 0.000\,35$ days. The time in days should therefore be quoted to no more than 4 decimal places.

Question 7

The formula for cell G10 is `=E10+F10`. The value should be 0.7903.

Question 8

The typical uncertainty, which applies to most of the data, is ±0.04 magnitudes (column D). This is likely to be due to the limitations of the telescope and detector used for the observations under good observing conditions, so cannot be improved upon without a bigger telescope or more sensitive detector. The larger values are likely to be due to external factors, such as poor weather.

Question 9

You should obtain a value close to 7.6 days. Anything between 7.4 and 7.8 looks reasonable so the uncertainty is ±0.2 days. Your folded light curve after selecting the best period should look like Figure 9.

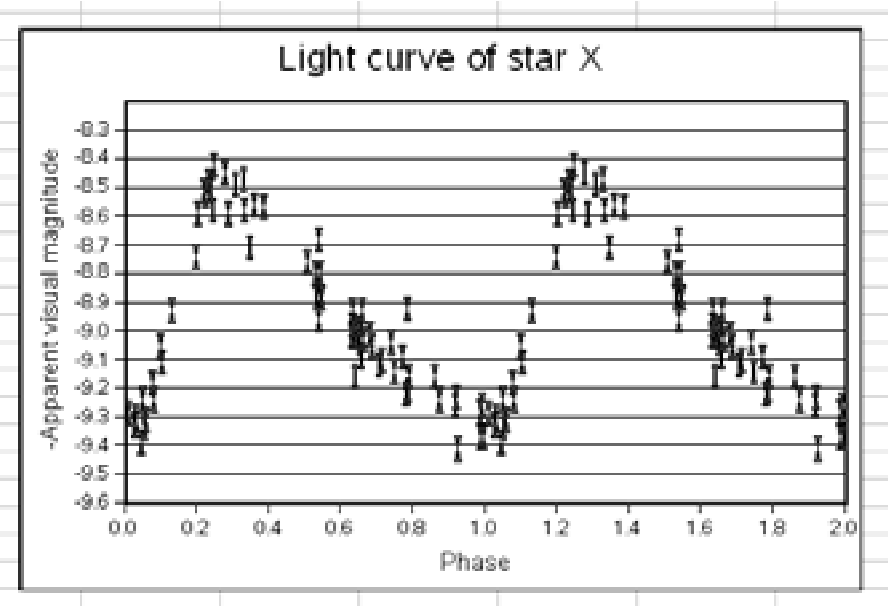


Figure 9 The final folded light curve of star X for a period of 7.6 days.

Question 10

Substituting P = 7.6 days into Equation 4 gives an absolute visual magnitude of

$$M_V = -2.8 \log(7.6) - (1.4) = -3.87$$

Applying the uncertainty in P gives

$$M_{V(max)} = -2.8 \log(7.6 + 0.2) - (1.4) = -3.90$$

$$M_{V(min)} = -2.8 \log(7.6 - 0.2) - (1.4) = -3.83$$

So the uncertainty in the magnitude resulting from the period uncertainty is approximately ±0.04.

Combining this with the uncertainty in the period–luminosity relation (i.e. ±0.2)

$$\Delta M_V = \sqrt{[(\Delta M_{V(\text{period})})^2 + (\Delta M_{V(\text{PL relation})})^2]}$$
$$= \sqrt{[(0.04)^2 + (0.2)^2} = 0.20$$

(The uncertainty resulting from the period is negligible compared with the scatter in the period–luminosity relation.)

The absolute visual magnitude of star X is therefore $M_V = -3.87 \pm 0.20$.

(Note that if this were the final answer you would probably quote it as $M_V = -3.9 \pm 0.2$. However, it is best to retain an extra decimal place during the calculations to prevent rounding errors. The final answer, in this case the distance, can then be given to the appropriate number of significant figures.)

Question 11

The approximate maxima and minima of the light curve are 8.45 and 9.35. The mean apparent visual magnitude is therefore 8.9 with an estimated uncertainty of ±0.05 based on the scatter in the points.

Question 12

Rearranging Equation 2, $M_V = m_V - 5 \log d + 5$, gives

$$d = 10^{(m_V - M_V + 5)/5}$$

$$d = 10^{(8.9 + 3.87 + 5)/5} = 3581\ \text{pc}$$

The uncertainty in the distance is a result of the uncertainty in $m_V - M_V$. The uncertainty in this quantity is

$$\sqrt{[(0.05)^2 + (0.20)^2]} = 0.21$$

$$m_V - M_V = 12.77 \pm 0.21$$

The resultant maximum and minimum likely values of d are therefore:

$$d_{max} = 10^{(12.77+0.21+5)/5} = 3945\ \text{pc}$$

$$d_{min} = 10^{(12.77-0.21+5)/5} = 3251\ \text{pc}$$

The uncertainty in d is therefore approximately ±400 pc.

The distance to star X is therefore $d = (3600 \pm 400)$ pc.

Question 13

It has been assumed that there is no interstellar absorption.

S282 Astronomy

Investigating supernova remnants

Study time: 90 minutes

Summary

In this activity you will be using images of the remnants of four comparatively recent supernovae: Cassiopeia A (Cas A), the Crab, Kepler, and Tycho. You will investigate the supernova remnants (SNRs) at radio, optical and X-ray wavelengths, observe the similarities and differences, and use the on-screen ruler to measure the angular sizes of the SNRs and calculate their mean expansion speeds.

You should have read Sections 8.3 and 8.4 of *An Introduction to the Sun and Stars* before starting this activity.

Learning outcomes

- Familiarity with the morphologies of supernova remnants at radio, optical and X-ray wavelengths.
- Calculate the radius of an astronomical body given its angular size and distance.
- Calculate the time-averaged expansion speed of a supernova remnant.
- Recognize that the time-averaged expansion speed of a supernova remnant is an overestimate of the current expansion speed.
- Understand that spatial resolution depends on the telescope used, and that some differences in appearance between images taken in different wavelength bands arise from differences in spatial resolution.

Background to the activity

The supernova that gave rise to the Cassiopeia A SNR is thought to have exploded about 300 years ago, though there are no records of it being observed. In the activity you will find a possible explanation for why this supernova was not seen at the time.

The Crab, Tycho and Kepler SNRs are all remnants of supernovae that were observed when they exploded; the Crab by Chinese astronomers and Tycho and Kepler by the European astronomers whose names they now bear. The Crab supernova remnant is the most easily viewed supernova remnant in the sky – it can be seen using large binoculars or a small telescope.

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Some basic data about the four SNRs that you will study in this activity are given in Table 1.

Table 1 Supernova remnant data.

	Cas A	**Crab**	**Tycho**	**Kepler**
year of supernova (AD)	about 1700	1054	1572	1604
distance[1]/kpc	3.4	1.9	2.4	5.0
RA	23 h 23 min 24 sec	05 h 34 min 32 sec	00 h 25 min 13 sec	17 h 30 min 41 sec
Dec	+58°49′	+22°01′	+64°09′	−21°29′
constellation	Cassiopeia	Taurus	Cassiopeia	Ophiuchus

[1]Note that the distances to supernova remnants are very difficult to determine with any certainty. For the purposes of this activity we will assume (rather simplistically) that all of the distances quoted here are known to an accuracy of ±20%.

Part 1 Multiwavelength views of supernova remnants

You will be using optical, X-ray and radio images of the four supernova remnants in Table 1. The optical data are from the Digitized Sky Survey, which is sensitive to a wavelength range that is centred on $\lambda \approx 500$ nm. Note that these data are shown as photographic negatives – the sky is white and any sources of emission are grey or black. The X-ray data are from an instrument called the High Resolution Imager (HRI) on the ROSAT observatory – these maps are sensitive to photons with energies of about 1 keV (i.e. 10^3 eV) or equivalently $\lambda \approx 1$ nm. The radio data are from a survey carried out by the Very Large Array (VLA) that is sensitive to radio waves with wavelengths $\lambda \approx 0.2$ m.

The year in which the observations were made are given in Table 2.

Table 2 The dates of observation of the multiwavelength image set of supernova remnants.

Band	**Year of survey**
optical	1956, except Kepler SNR (1987)
X-ray	1992
Radio	1995

All images are provided in the Image Archive.

- Start the S282 Multimedia guide and open the folder called ‘Stars’, then click on the icon for this activity (‘Investigating supernova remnants’).
- Press the Start button to launch the Image Archive at the required set of images.
- Alternatively, launch the Image Archive by clicking the Image Archive button in the Multimedia guide and then find the ‘Supernova Remnants – Multiwavelength images’ set which is located within the ‘Stars’ section of the archive.

Once you have opened the Image Archive at the page that displays the multiwavelength images of supernova remnants take a few moments to read the instructions of how to display the different images. You are now going to look at the images in each wavelength band (optical, X-ray and radio) for each supernova remnant in turn.

- Click on the button labelled Cas A SNR that is on the upper right-hand part of the screen.
- An optical image of the field of view that contains the Cassiopeia A supernova remnant should now be displayed. Don't be too concerned if you can't work out where the supernova remnant is! You are now going to look at the same field of view in the X-ray and radio parts of the electromagnetic spectrum.
- At the lower right-hand part of the screen you should see a table that has three columns that are headed Image, Contour and Contour. To start with, you are simply going to look at images at different wavelengths, and these are accessed by clicking on the relevant wavelength range in the Image column.
- Click on the term X-ray in the Image column. This will display an X-ray image of the supernova remnant. Note what you see and how it compares to the optical image.
- Now view the radio image by clicking on the term Radio in the Image column. A radio image of the supernova remnant will be displayed. Again, note how it compares to the optical image.

View the optical, X-ray, and radio images for each of the four supernova remnants in turn. As you do so, make notes of appearance of the supernova remnant at this waveband and attempt to answer the following questions.

Question 1

(a) Is every supernova remnant visible in all three wavelength ranges (optical, X-ray and radio)?

(b) Which wavelength ranges would be best suited to searching for supernova remnants?

In answering Question 1 you probably came to the conclusion that the optical images are not well suited to the detection of supernova remnants.

- Suggest a reason why not all supernova remnants are visible at optical wavelengths.

❑ The primary reason is due to the absorption of visible light by dust in the interstellar medium between us and the remnant.

This is most clearly seen in the case of the Cas A remnant. Some of the remnant is visible at optical wavelengths, but most appears to be missing. This suggests that something – interstellar dust – is obscuring the major part of the remnant. This also offers an explanation as to why the supernova that gave rise to Cas A was not seen – it was most probably obscured by the intervening interstellar medium.

Question 2

Compare the X-ray images of all four supernova remnants with one another.

(a) Are any of the images similar, and if so, in what way are they similar? What does the appearance of these remnants suggest about their structure?

(b) One of the supernova remnants is distinctively different to the other three. Identify this remnant and describe in what way it looks different.

You are now going to compare emission at two different wavelengths. A common technique in astronomy is to show emission at two different wavelengths by overlaying an image at one wavelength with a contour map that corresponds to the other wavelengths. To start with, however, you will see how a contour map relates to one of the images that you have already viewed.

- For the Cas A supernova remnant view the X-ray image by clicking on the term X-ray in the Image column.
- Now display the X-ray image with X-ray contours by clicking on X-ray in the same row of the table as before but now in the column marked Contour.

You should be able to see that the contours simply follow the intensity of the X-ray emission from the supernova remnant. Just as contours on a map indicate the topography with lines of equal height, these contours show lines of equal surface brightness. Now you are going to compare the X-ray emission with the optical image of the remnant.

- In the row of the table that starts with the term Optical, click on the term X-ray that occurs in the Contour column. You should see the optical image with the X-ray contours overlaid.

Question 3

Is there any correspondence between the optical and X-ray emission of the Cas A supernova remnant?

Now that you have seen one combination of an image with a contour overlay try other combinations and for different SNRs. When you have understood how to show any combination of image and contours that is available, attempt the following question.

Question 4

Compare the X-ray emission with the radio emission for all four supernova remnants. For each case describe whether the X-ray emission and the radio emission seem to arise from the same parts of the supernova remnant. Also make a note of any general differences between the images in these two wavelength bands.

In answering Question 4 you may have noticed that the radio emission appears to be more smoothly distributed than the X-ray emission, and that it appears to have a greater extent.

- ■ Give two possible reasons for the radio emission appearing to be more smoothly distributed than the X-ray emission.
- ❑ One cause of this difference may be that the radio and X-ray emitting regions are genuinely different and distributed as shown.

  Another cause of this difference may be that the spatial resolution of the X-ray image is different to that of the radio map. It could be that both the X-ray and radio emitting gas are distributed in the same way, but that radio images are more 'blurred' because the spatial resolution is lower.

In fact, there is a substantial difference between the spatial resolution of the images at different wavelengths. The X-ray images have a spatial resolution of about 7 arcsec, whereas the radio maps have a spatial resolution of 45 arcsec. Any features in an image that have an extent that is less than the resolution of that image will become spread out and appear about the same size as the spatial resolution. One important implication of using a low-resolution map to study a supernova remnant is that this 'blurring' will make the remnant appear more spread out than it really is.

Thus the most likely reason for the radio emission appearing to be more smoothly distributed and spread out than the X-ray emission is due to the different spatial resolution of the maps in these two wavelength bands. In fact, higher resolution radio maps confirm this view.

It should be noted that radio maps can be made over a wide range of resolutions that depend on the way in which a radio telescope (or a group of radio telescopes) is used. You may see examples of radio maps that have resolutions ranging from a few milli-arcseconds up to a few degrees – depending on the instruments and techniques adopted. The radio data shown here are from a survey of the sky that was carried out at a fairly low spatial resolution.

Part 2 Measuring the time-averaged expansion speeds of supernova remnants

In this part of the activity you will measure the mean speed of expansion of each supernova remnant by measuring how far the outer edge of the remnant has travelled since the time of the initial explosion. This will involve measuring the angular size of each remnant and then calculating their radii. However, the first task is to decide what images you are going to use for your measurements, and this is the subject of the next question.

Question 5

What criteria are important in deciding which wavelength band (optical, X-ray or radio) should be used for determining the angular size of the supernova remnants? Which wavelength band (optical, X-ray or radio) should be used to determine the angular sizes of each of the remnants?

In answering Question 5 you should have concluded that in the case of the Crab it is the optical image that should be used to determine the angular extent of the remnant. For the other three supernova remnants it is the X-ray images that should be used for this measurement.

It should be noted however that there is an inherent danger in using different wavelength ranges to measure what you assume to be the same type of feature in different astronomical objects. It may be that the features at different wavelengths are profoundly different in some way, and could lead to quite erroneous conclusions. Having said that, we shall carry on with the analysis on the basis described above, but you should keep in mind that the Crab supernova remnant is being measured in a different way to the others.

Measuring the angular diameter

- Select the Tycho SNR and look at the X-ray image overlaid with the X-ray contours.

This provides a good picture of the expanding spherical shell of material. The shock wave is travelling so fast that the surrounding interstellar medium (ISM) has no warning of what is about to hit it. The outermost contour gives a reasonable estimate of the position of the shock front.

To help you to measure the angular diameter of these images the Image Archive includes a screen ruler. This is a tool that gives the angular distance in arc minutes between any two positions on the image. The ruler is used as follows:

- Place the cursor on the left-hand limb of the remnant. Now, while holding down the left mouse button, drag the cursor from this position across the image to the right-hand limb. As you do so you should see a line stretch out between the position that you started from and the current cursor position. Release the mouse button when you have positioned the cursor on the right-hand limb.

In addition to the line between the two points you should also see a white panel that gives the length of the line in terms of the pixels of the image and the angular distance covered by the line in units of arcmin. It is the angular distance given in arcmin that we shall use for our measurements here. (This information is also given at the top of the screen, just beneath the title.)

- Try measuring the diameter of the supernova remnant in several different directions (left to right, top to bottom, etc.). Record your measurements and estimate the angular diameter α of the remnant. Insert this value of α – expressed to an appropriate number of significant figures – into Table 3. Note: the next few sections take you through the process of completing Table 3.

Question 6

(a) By using several measurements, obtain a value for the diameter of the supernova remnant.

(b) Try to make a (rough) estimate of the uncertainty in this measurement. Hence quote the mean angular diameter of the supernova remnant with its associated uncertainty.

(c) Express the estimated uncertainty as a relative uncertainty.

(d) What is the origin of the uncertainty?

Table 3 Table of results for determining the time-averaged expansion speeds of supernovae.

	Cas A	**Crab**	**Tycho**	**Kepler**
Wavelength band used for measurement of angular diameter				
Angular diameter α/arcmin (from your measurements)				
Angular radius (α/2)/radians				
Distance d/pc (from Table 1)				
Radius R/pc				
Radius R/km				
Date of supernova explosion (from Table 1)				
Date of observation (from Table 2)				
Time since explosion t/s				
Time-averaged expansion speed and associated uncertainty υ/km s^{-1}				

Finding the radius of the supernova remnant

We use the same approach as in *An Introduction to the Sun and Stars* Section 3.3.1 for measuring the radius of a star:

$$R = [(\alpha/2)/\text{radians}] \times d$$

(*An Introduction to the Sun and Stars* Equation 3.8)

where d is the distance to the SNR, α is its angular diameter (so $\alpha/2$ is its angular radius), and R is its radius in the same units as d. Note that the angle must be expressed in radians rather than in arcmins.

There are two measured quantities, α and d, in the above equation, and both of these quantities have associated uncertainties. Strictly speaking the uncertainty in R should be calculated by combining the uncertainties in both α and d. However, in your answer to Question 6 you should have found that the relative uncertainty in α is only a few per cent, whereas the relative uncertainty in d is about 20% (see the footnote to Table 1). It is the uncertainty in d that is the most important factor in determining the uncertainty in R. In fact, it is reasonable to assume that the relative uncertainty in R is the same as the relative uncertainty in d, i.e. the calculated radius will also have an uncertainty of ±20%.

Question 7

Using the angular radius that you have just measured for the Tycho supernova remnant and data from Table 1, calculate the radius R of the supernova. Express this radius in parsecs and km. Also calculate the uncertainty in the calculated radius. Enter your results in Table 3.

(You may assume 1 arcmin = 2.91×10^{-4} radians, and 1 parsec = 3.09×10^{13} km.)

Determining the time-averaged expansion speed

The time-averaged expansion speed of the shock front can be found by dividing the radius R of the remnant by its age t

$$v = R/t$$

As far as the uncertainty in v is concerned, it can be assumed to be the same as the relative uncertainty in R, i.e. ±20%. (This is because the relative uncertainty in the time elapsed between the supernova explosion and the date of the observation is small in comparison to the relative uncertainty in R.)

Question 8

Calculate the time-averaged speed of expansion of the Tycho SNR in km s^{-1}. Express this value and its uncertainty to an appropriate number of significant figures.

Comparing the time-averaged expansion speeds of supernova remnants

The final part of the activity involves calculating the time-averaged expansion speeds of the other three supernova remnants (Cas A, Crab and Kepler).

Question 9

(a) Determine the mean expansion speeds for the other three SNRs in the same way and complete Table 3. (You may want to write a spreadsheet to perform these calculations.) Note that rather than repeating the uncertainty calculations it is sufficient to assume that the relative uncertainty of all of the calculated time-averaged expansion speeds is ±20%.

(b) How do the expansion speeds compare with one another?

As a result of your calculations you should have found that the time-averaged expansion speeds of these supernova remnants lie in a range of between about 1×10^3 and 8×10^3 km s^{-1}. Even by astronomical standards these are high speeds, and are an indication of the vast amount of energy that was released in the initial supernova explosion. You should also have noticed that the time-averaged expansion speed of the Crab remnant is significantly lower than the expansion speeds of the other three remnants.

We end this activity by considering whether this difference in expansion speeds is a real effect and, if so, whether it has a straightforward explanation.

■ Is there any difference in the method you used to determine the expansion speeds between the Crab remnant and the other three remnants?

❑ Yes. The expansion speed of the Crab remnant was measured from the optical image, whereas the expansion speeds of the other three remnants were determined from X-ray images.

This type of problem can be addressed by measuring the expansion speed of the Crab at X-ray wavelengths. A quick inspection of the X-ray image shows that the Crab remnant appears smaller in the X-ray images than it does in the optical – so this would imply an expansion speed that is even smaller than the value we calculated from the optical image. So it appears that this is a real effect and not one that has arisen from the way in which we have analysed the data.

Question 10

Do you think the speeds you have found are the current expansion speeds? Justify your answer.

You might think that the low expansion speed of the Crab remnant is due to its age – it is over twice the age of the other remnants in our sample. However, detailed analysis shows that this is not the case: even when age is accounted for, the Crab remnant appears to be expanding at an anomalously slow rate. This implies that the supernova that gave rise to the Crab remnant probably released less energy than is typical.

You have already noticed that the Crab remnant looks quite different to other supernova remnants, but the reasons for this and for its slow expansion rate are a mystery. So the most famous and most easily visible supernova remnant is, ironically, one of the least understood.

Answers to questions

Question 1

(a) Table 4 provides a summary of whether individual supernova remnants can be seen at different wavelengths. It can be seen that these supernova remnants are not all visible at radio wavelengths, whereas all are visible at optical and X-ray wavelengths.

Table 4 The visibility of supernova remnants at different wavelengths.

	Cas A	**Crab**	**Tycho**	**Kepler**
Optical	partly visible	yes – clearly visible	no	no
X-ray	yes	yes	yes	yes
Radio	yes	yes	yes	yes

(b) On the basis of these images it would seem that searches for supernova remnants would best be conducted in the radio or X-ray parts of the spectrum.

Question 2

The X-ray images of the Tycho, Kepler and Cas A supernova remnants all have a similar appearance. These three images show emission from roughly circular region, and this emission is concentrated towards the circumference of the circle. The fact that all three of these SNRs look circular from Earth is an indication that the general morphology is close to spherical – it is unlikely that we would be at

just the right angle to see each of them as a circle otherwise. Furthermore the appearance of each image also seems to suggest that emission comes from a roughly spherical shell – towards the edge of the remnant we look though a greater depth of this shell and this gives rise to the bright edges.

The X-ray image of the Crab supernova remnant is quite different to the other three. In this case there is a very bright region of X-ray emission towards the centre of the remnant. There is no obvious 'shell' appearance to the remnant.

(Comment: a distinctive feature of the Crab is the presence of a pulsar near the centre of the remnant. It is thought that this pulsar is an energy source to its environment and causes the bright regions of X-ray and radio emission that are seen here. Pulsars are considered in more detail in Chapter 9 of *An Introduction to the Sun and Stars*.)

Question 3

There is some optical emission that corresponds to the uppermost part of the remnant as viewed in X-rays. However, other parts of the supernova remnant that are bright in X-rays are not visible in the optical image.

This arises primarily because there are differing amounts of interstellar dust along different lines of sight to the remnant. Thus, in at optical wavelengths, the uppermost part of the remnant suffers from less absorption than the rest of the remnant.

Question 4

The X-ray and radio emission can be compared by examining either the X-ray image with radio contours or the radio image with X-ray contours (or both). The similarities and differences between the emission in these wavelength bands are:

Cas A, Kepler and Tycho SNRs

1 The roughly spherical shell of emission is visible in both radio and X-rays.

2 The locations that show the most intense X-ray emission do not correspond to the regions that show the brightest radio emission.

3 The distribution of the radio emission appears smoother than that of the X-ray emission, and appears to have a greater extent.

Crab SNR

1 The radio and X-ray emissions appear to be quite different in extent. (The radio emission follows the optical emission quite well, but the X-ray emission is much more centrally concentrated.)

2 There is a bright spot of emission in both the X-ray and the radio emission – these are in roughly, but not exactly, the same location.

3 As is the case for the other three remnants, the distribution of the radio emission appears smoother and appears to have a greater extent than does that of the X-ray emission.

(Comment: the point about the radio emission appearing to be more smoothly distributed than the X-ray emission is taken up in the text immediately following Question 4 on pp. 4–5.)

Question 5

The most important criterion is that the remnant should be clearly visible! Thus observations at optical wavelengths are ruled out for Kepler, Tycho and Cas A, and either radio or X-ray images should be used for measuring the diameter of these remnants. On this basis, any of the three wavelength bands could be used for measuring the angular extent of the Crab supernova remnant.

A second criterion, that was hinted at the end of Part 1, is that we should use the highest resolution images available. This will allow the edge of the remnant to be accurately located. In the case of the Crab remnant it is the optical image that provides the highest resolution and so this is the image that should be analysed. In the other three remnants we have already seen that there is a choice between using radio and X-ray images. However, in the notes it is stated that the X-ray images are at higher resolution than the radio images, and so it is the X-ray images that should be used.

Question 6

(a) I measured the diameter of the supernova remnant in four directions and obtained the following values:

Direction	**Angular diameter/arcmin**
vertical	8.1
horizontal	8.0
diagonal – upper left to lower right	8.2
diagonal – upper left to lower right	7.5

The mean value of the angular diameter is then

$\alpha = (8.1 + 8.0 + 8.2 + 7.5)/4$ arcmin = 7.95 arcmin

(b) The four measured values differ by a few tenths of an arc minute in each case, so a reasonable rough estimate of the uncertainty would be between about 0.2 and 0.5 arcmin. Here we will use a value of 0.3 arcmin.

The mean angular diameter with associated uncertainty is then
$\alpha = (8.0 \pm 0.3)$ arcmin

(c) The relative uncertainty is

$\Delta\alpha / \alpha \approx 0.3/7.95 = 0.038$

So the relative uncertainty in the measurement of the diameter of the supernova remnant is about 4%. (If you obtained a value of between 2% and 6% that would be a reasonable estimate.)

(d) The source of this uncertainty is not in the operation of the ruler but in the fact that the remnant is not perfectly spherical!

Question 7

The angular diameter I measured (see the answer to Question 6(b)) for the Tycho supernova remnant was 8.0 arcmin. The first step is to convert this into an angle in radians

$\alpha = 8.0 \text{ arcmin} = 8.0 \times 2.91 \times 10^{-4} \text{ rad} = 2.33 \times 10^{-3} \text{ rad}$

The radius of the remnant is found using

$R = [(\alpha/2)/\text{radians}] \times d$

(*An Introduction to the Sun and Stars* Equation 3.8)

In this case $d = 2.4$ kpc $= 2.4 \times 10^3$ pc (from Table 1), so

$R = (2.33 \times 10^{-3}/2) \times 2.4 \times 10^3$ pc

$R = 2.79$ pc

Expressed in terms of km this is

$R = 2.79 \times 3.09 \times 10^{13}$ km $= 8.63 \times 10^{13}$ km

As stated in the text, the relative uncertainty in R is the same as the relative uncertainty in d.

$\Delta R/R = 0.20$

$\Delta R = 0.20 \times R = 0.6$ pc $= 1.7 \times 10^{13}$ km

So the radius of the supernova remnant is

$R = (2.8 \pm 0.6)$ pc $= (8.6 \pm 1.7) \times 10^{13}$ km

Question 8

To calculate the time-averaged expansion speed it is necessary to know the radius of the supernova remnant and the time elapsed between the supernova explosion and the observation.

From Question 7 the radius is

$R = (8.6 \pm 1.7) \times 10^{13}$ km

The time elapsed between the year of the Tycho supernova explosion (1572, see Table 1) and the date of the X-ray observation (1992, see Table 2) is

$t = (1992 - 1572) \times 365 \times 24 \times 60 \times 60$ s $= 1.32 \times 10^{10}$ s

The time-averaged expansion speed is

$\upsilon = R/t = (8.6 \times 10^{13})$ km$/(1.32 \times 10^{10})$ s

$= 6.52 \times 10^3$ km s^{-1}

The relative uncertainty in this value is ±20%, so

$\Delta\upsilon = 0.20 \times 6.52 \times 10^3$ km s^{-1}

$= 1.30 \times 10^3$ km s^{-1}

So the final result is that time-averaged expansion speed as measured from the X-ray image is

$\upsilon = (6.5 \pm 1.3) \times 10^3$ km s^{-1}

Question 9

(a) The method for calculating the time-averaged expansion speeds of the other remnants is the same for Tycho. The only problem of note is that the Crab remnant is clearly not circular, but an estimate of the mean diameter can still be made by measuring the remnant along its long and short axes.

Table 5 shows one set of results and calculations. Note that the intermediate values used in calculating the final result are quoted to one significant figure more than is justified by the data – this is to reduce rounding errors in the intermediate calculations. Also note that your values may differ from those given here, but you should arrive at values of the time-averaged expansion speed that are consistent with those quoted here.

Table 5 A completed table of results on the time-averaged expansion speed of supernova remnants.

	Cas A	**Crab**	**Tycho**	**Kepler**
Wavelength band used for measurement of angular diameter	X-ray	optical	X-ray	X-ray
Angular diameter α/arcmin (from your measurements)	4.7	4.3	8.0	3.5
Angular radius (α/2)/radians	6.84×10^{-4}	6.26×10^{-4}	1.16×10^{-4}	5.09×10^{-4}
Distance d/kpc (from Table 1)	3.4	1.9	2.4	5.0
Radius R/pc	2.33	1.25	2.79	2.55
Radius R/km	7.18×10^{13}	3.87×10^{13}	8.63×10^{13}	7.87×10^{13}
Date of supernova explosion (from Table 1)	about 1700	1054	1572	1604
Date of observation (from Table 2)	1992	1956	1992	1992
Time since explosion t/s	9.21×10^{9}	2.84×10^{10}	1.32×10^{10}	1.22×10^{10}
Time-averaged expansion speed and associated uncertainty υ/km s^{-1}	$(7.8 \pm 1.6) \times 10^{3}$	$(1.4 \pm 0.3) \times 10^{3}$	$(6.5 \pm 1.3) \times 10^{3}$	$(6.4 \pm 1.3) \times 10^{3}$

(b) From Table 5 is can be seen that Cas A, Tycho and Kepler all have similar mean expansion speeds of between 6×10^3 and 8×10^3 km s^{-1}, whereas the expansion speed of the Crab is less than 2×10^3 km s^{-1}.

Question 10

As the SNR expands it slowly begins to cool by radiating energy away, and it also sweeps up additional material from the surrounding interstellar medium, so the kinetic energy of the individual particles decreases. This means that the speed will gradually decrease, so the current speed will be less than the mean speed since the explosion.

S282 Astronomy

Observing the energetic Universe

Study time: 1 hour

Summary

This web-based activity has three parts. In the first you will answer some questions about X-ray astronomy using the website devoted to the Chandra X-ray observatory. Following this you will compare two X-ray observatories – Chandra and XMM-Newton. Finally, you will use images taken with these observatories to explore the findings that link pulsars to supernova remnants.

This activity will require you to connect to the Internet for the whole session – about an hour.

You should have read to the end of Section 9.5 of *An Introduction to the Sun and Stars* before starting this activity.

Learning outcomes

- Appreciate the importance of X-ray astronomy in understanding a wide variety of energetic processes in the Universe.
- Identify, compare, evaluate and synthesize information from web-based resources.

Background to the activity

Over the last 40 years or so X-ray astronomy has been developed into an extremely important window on the Universe. This has been marked by the award of the Nobel Prize for Physics for 2002 to Riccardo Giacconi, one of the pioneers of the field. The technological development of this new area of astronomy has been extremely rapid and has greatly improved our understanding of some of the most energetic and violent processes in the Universe.

Question 1

(a) What are the typical wavelengths of X-rays?

(b) What are the typical photon energies of X-rays? Express your answer in electronvolts.

As you found in Question 1, the typical photon energies of X-rays are of order 10^3 eV. You will see that in X-ray astronomy it is common to describe photon energies in units of keV (1 keV = 10^3 eV).

Question 2

Calculate the temperature of an object that radiates with a black-body spectrum that peaks at a photon energy of 1 keV.

In this activity we will investigate the importance of X-ray astronomy in understanding a wide variety of energetic phenomena by considering two current generation X-ray satellites: Chandra and XMM-Newton.

Part 1 X-ray astronomy

- Start your web browser and connect to the Internet.
- Open the Chandra website (http://chandra.harvard.edu/). The first part of the activity will consist of answering a number of questions about X-ray astronomy using information that is provided on this website.
- Towards the top of the screen on the Chandra home page you should see a link called Field Guide. Follow this link and take a few minutes to see what sort of information is provided. Most of the information that you will need to answer Questions 3 to 5 can be found by following links from the Field Guide page.

Question 3

This question is about the information that X-rays can provide about stars and stellar remnants.

(a) Are X-rays produced in significant quantities by main sequence stars?

(b) Which types of stellar remnant are likely to produce X-rays? Under what circumstances are such objects very luminous X-ray sources?

Question 4

How can X-ray spectroscopy help us to determine the chemical composition of emitting regions?

Part 2 X-ray observatories

So far we have considered why we want to detect cosmic X-rays but not how such observations are made. In this part of the activity we will look at how X-ray astronomy is carried out.

- From the Chandra home page follow the link to Field Guide, and then study the links under the X-ray Astronomy heading. Spend a few minutes reading the information on these pages to get a feel for what is involved in making X-ray observations. In particular, you should study the X-ray Absorption page.

Consider the following in light of what you have read on these pages:

- Why do we need to use a space-based telescope to detect cosmic X-rays?

❑ The Earth's atmosphere is opaque to X-rays and so our X-ray telescope must be taken above the bulk of the Earth's atmosphere.

In practice this means using a satellite, although early X-ray detectors were flown in sounding rockets and balloons. Rockets and balloons are still frequently used to test new types of X-ray telescope since the costs of launching a payload in this way are much less than a satellite launch, but the time above the atmosphere is very limited (minutes for a sounding rockets and days for a balloon).

Another aspect of X-ray astronomy that distinguishes it from optical astronomy relates to the techniques used to collect and focus X-rays.

- From the Chandra home page follow the link to Field Guide and then to History of X-ray Astronomy. Read the information about the history of X-ray astronomy, and then attempt the following question.

Question 5

Briefly describe how an X-ray telescope focuses X-rays.

At present there are two major X-ray satellites in operation with good imaging characteristics – Chandra (operated by NASA) and XMM-Newton (operated by the European Space Agency). In some ways these are similar instruments but in other ways their capabilities complement one another. In the next question you will use the Chandra and XMM-Newton websites to compare these two instruments.

- For this exercise it is probably best to use two browser windows for ease of comparison.
- From the Chandra website select the About Chandra link from the top of the screen.
- Open a new browser window (press Ctrl-N to open another window) and go to the XMM-Newton home page (http://sci.esa.int/xmm/).

Question 6

(a) Navigate around the Chandra and XMM-Newton websites to find the information required to complete Table 1.

(b) Using the information that you obtained in part (a) describe the aspects of Chandra and XMM-Newton that are similar, and those that are substantially different.

(c) Which observatory would be best suited to: (i) making detailed images of X-ray emitting sources, and (ii) observing the X-ray spectrum of faint sources?

Table 1 Information about the Chandra and XMM-Newton X-ray telescopes (for use with Question 4).

Feature of telescope	**Chandra**	**XMM-Newton**
Focal length/m	10	
Angular resolution/arcsec		
Collecting area (at a photon energy of about 1 keV)/cm^2		
Range of photon energies that can be detected/keV		

Both observatories have a number of instrument packages that can be used with the respective telescopes. There are two main types of instrument that are employed on both – imaging cameras and spectrometers. Chandra employs the High Resolution Camera (HRC) while XMM-Newton uses the European Photon Imaging Camera (EPIC).

Question 7

What is the essential difference between the imaging cameras on the two observatories?

Part 3 Pulsars and supernova remnants

One of the major achievements of the Chandra mission has been to help make the link between pulsars and supernova remnants. Although our ideas about pulsars suggest that these objects should be born in supernovae, before the arrival of Chandra there was little direct evidence for their unambiguous association. The Crab and Vela were the most obvious examples of pulsars associated with supernova remnants, but there were few others. There are two reasons for this:

1. The supernova is likely to 'kick' the pulsar away from the site of the explosion. Thus, only comparatively young pulsars are likely to be found within the supernova remnant.
2. Young supernova remnants and pulsars are not particularly luminous in the wavebands where imaging could be done efficiently pre-Chandra (e.g. optical, radio) since they are still at high temperatures. The good angular resolution of Chandra means that we can now take images in X-rays with high angular resolution to reveal the detailed structure of young supernova remnants.

In the final part of this activity you will use images taken with Chandra to show the association between pulsars and supernova remnants.

- Return to the Chandra home page and follow the links to Photo Album, Chandra Images by Category, e.g. Supernovas (or Supernovas & SNR) and Neutron Stars (or Neutron Stars/X-ray Binaries).

Question 8

The Crab Nebula surrounds the pulsar that remains from the supernova that produced it. Find two further associations between neutron stars and supernova remnants that have been observed by Chandra Are all of the X-ray sources that are thought to be neutron stars observed as pulsars?

Endnote

In this activity we have explored the importance of X-ray astronomy in telling us about energetic systems. We have also investigated the two premier X-ray observatories currently operating, including a brief look at how they compare. Finally, we looked at one of the questions that X-ray astronomy may be able to answer – how pulsars relate to the supernovae that formed them.

This completes our very quick tour of the energetic Universe using Chandra and XMM-Newton. Many of these topics are explored in much greater detail in the Open University Level 3 physics and astronomy courses.

Answers to questions

Question 1

(a) Figure 1.36 (of *An Introduction to the Sun and Stars*) shows that X-rays lie in the electromagnetic spectrum between the ultraviolet and γ-ray regions with wavelengths in the range 3×10^{-9} to 3×10^{-11} m.

(b) The energy of a photon is given by *An Introduction to the Sun and Stars*, Equation 1.3

$$E = hf$$

Where f is the frequency of the electromagnetic radiation and h is the Planck constant. The frequency of electromagnetic radiation is related to wavelength λ by *An Introduction to the Sun and Stars* Equation 1.2, which can be rearranged to give

$$f = c/\lambda$$

These two equations can be combined to give an equation for photon energy in terms of wavelength

$$E = hc/\lambda \qquad \text{(Equation A)}$$

Thus a photon with wavelength $\lambda = 3 \times 10^{-9}$ m has an energy of

$$E = (4.14 \times 10^{-15}\ \text{eV s}) \times (3.00 \times 10^{8}\ \text{m s}^{-1})/(3 \times 10^{-9}\ \text{m}) = 4 \times 10^{2}\ \text{eV}$$

and a photon with wavelength $\lambda = 3 \times 10^{-11}$ m has an energy of

$$E = (4.14 \times 10^{-15}\ \text{eV s}) \times (3.00 \times 10^{8}\ \text{m s}^{-1})/(3 \times 10^{-11}\ \text{m}) = 4 \times 10^{4}\ \text{eV}$$

(Note that we have used the value of $h = 4.14 \times 10^{-15}$ eV s, so that the photon energy will come out in units of electronvolts as required.)

So X-rays typically have photon energies between a few 10^{2} eV and a few 10^{4} eV.

Question 2

The first stage is to find the wavelength that corresponds to emission at a photon energy of 1 keV. Equation A from Question 1 provides a relationship between photon energy and wavelength

$$E = hc/\lambda \qquad \text{(Equation A)}$$

This can be rearranged to give

$$\lambda = hc/E$$

In this case, $E = 1 \times 10^{3}$ eV, (and remembering to use a value of h in appropriate units), so:

$$\lambda = (4.14 \times 10^{-15}\ \text{eV s}) \times (3.00 \times 10^{8}\ \text{m s}^{-1})/(1 \times 10^{3}\ \text{eV}) = 1.2 \times 10^{-9}\ \text{m}$$

To find the temperature of a black-body source that peaks at a given wavelength we need to use Wien's displacement law (*An Introduction to the Sun and Stars*, Equation 1.4)

$$(\lambda/\mathrm{m}) = \frac{2.90 \times 10^{-3}}{(T/\mathrm{K})}$$

which can be rearranged to give

$$(T/\mathrm{K}) = \frac{2.90 \times 10^{-3}}{(\lambda/\mathrm{m})}$$

So for a photon with a wavelength of 1.2×10^{-9} m

$$(T/\mathrm{K}) = \frac{2.90 \times 10^{-3}}{1.2 \times 10^{-9}} = 2.4 \times 10^{6}$$

So the black-body source that peaks at a photon energy of 1 keV has a temperature of 2×10^6 K (to one significant figure).

Question 3

(a) The photospheres of main sequence stars typically have temperatures of a few times 10^3 K to a few times 10^4 K. We saw in Question 2 that X-rays are produced when matter is heated to $\sim 10^6$ K. This is a higher temperature than the photospheres of main sequence stars, so main sequence stars will not be copious producers of X-rays. Although the solar corona reaches temperatures of a few million K the X-ray luminosity produced is small because the corona is so diffuse. Similarly we would expect the emission from stellar coronae to be weak (see http://chandra.harvard.edu/xray_sources/normal_stars.html).

(b) The three types of stellar remnant that were introduced in Chapter 9 of *An Introduction to the Sun and Stars* are: white dwarfs, neutron stars and black holes. X-ray emission occurs in environments where the temperature is about 10^6 K. Typically such high temperatures are generated where high magnetic fields, extreme gravity or explosive forces occur. Stellar remnants can provide the conditions needed for X-ray emission.

Isolated white dwarfs have high surface temperatures and so may be sources of X-rays. However, such stars cool rapidly and do not remain as luminous X-ray sources for an appreciable time. Much more luminous X-ray sources are interacting binaries in which accretion occurs onto a white dwarf.

Some neutron stars are pulsars that are powered by the rotational energy of the star, and some of these are X-ray sources. Neutron stars may also exist in interacting binaries, and accretion onto a neutron star may result in a very luminous X-ray source.

Isolated black holes are not sources of X-ray emission, but interacting binaries in which accretion occurs onto a black hole are (like interacting neutron star binaries) strong X-ray sources.

In all three cases (white dwarf, neutron star and black hole) there is strong emission of X-rays in interacting binaries. Under such circumstances the X-ray emission can tell us something about the intense pressure and heating by compression and viscous (frictional) forces experienced by matter as it spirals into the stellar remnant. The brightest X-ray sources in our Galaxy are neutron stars and black holes and so X-rays provide an ideal tool for exploring these late stages of stellar evolution.

See also the following sites:

http://chandra.harvard.edu/xray_sources/white_dwarfs.html

http://chandra.harvard.edu/xray_sources/neutron_stars.html

http://chandra.harvard.edu/xray_sources/blackholes_stellar.html

Question 4

X-rays are also produced through emission and absorption by electrons bound in atoms. We have already seen (Section 1.3.2 of *An Introduction to the Sun and Stars*) that line spectra are produced when electrons change energy level in atoms; we have noted that the majority of such transitions are in the optical and UV regions of the spectra. However, in heavier atoms the innermost electrons are very tightly bound and transitions involving these electrons correspond to energies in the X-ray region. As with other spectral lines these transitions have characteristic energies that indicate which element is involved. Thus X-ray spectroscopy can identify the chemical composition of these very hot regions (see http://chandra.harvard.edu/xray_astro/xrays.html).

Question 5

X-rays cannot be focussed using the equivalent of lenses but instead a system of X-ray mirrors is used. This works because if X-rays strike a metal surface at a very small angle they will be reflected. By careful choice of the shape of these metal surfaces the X-rays can be focussed and an image formed. This type of X-ray telescope is known as a grazing incidence telescope (see http://chandra.harvard.edu/xray_astro/history.html).

Question 6

(a) A completed table of information about Chandra and XMM-Newton is given in Table 2.

Table 2 A summary of information about the Chandra and XMM-Newton X-ray telescopes.

Feature of telescope	Chandra	XMM-Newton
Focal length/m	10	7.5
Angular resolution/arcsec	0.5	5
Collecting area (at a photon energy of about 1 keV)/cm^2	400	4300
Range of photon energies that can be detected/keV	0.09–10	0.1–12

Note that the details for Chandra can be found in the ‘Chandra Specifications handout’ available from the observatory’s website. To access this information go to the About Chandra page of the site, click on Chandra Hardware in the left-hand menu and then follow this menu down to the bottom of the page. Here you will see a links to HTML and PDF versions of the handout.

The corresponding values for XMM-Newton can be found from the Fact Sheet link, which can be found on the menu on the left-hand side of the XMM-Newton home page.

(b) Both telescopes have similar focal lengths and operate over similar ranges of photon energy. The angular resolution of Chandra is about a factor of 10 better than that of XMM-Newton. However, the collecting area of XMM-Newton is more than a factor of 10 greater than that of Chandra.

(c) Since Chandra has the better angular resolution it would be better suited to making detailed images of X-ray emitting regions. The spectrum of a faint X-ray source would be better obtained using XMM-Newton since it has a larger collecting area than Chandra.

Question 7

The main difference is the technology used to detect the imaged X-rays. The EPIC imaging device on XMM-Newton uses charge-coupled devices (CCDs) that are essentially similar to the chips used in ordinary digital cameras (see the Instruments link on the menu on the left-hand side of the XMM-Newton home page). The main imaging instrument on Chandra is the High Resolution Camera (HRC). This instrument uses devices called micro-channel plates, which have higher spatial resolution than CCDs and so match the imaging capability of its telescope (see the Science Instruments link on the left of the About Chandra web page). High-energy X-ray photons liberate electrons from the front of a micro-channel plate. The electrons are accelerated along collimated channels to generate a signal from which the arrival directions and energies of the original X-ray photons can be measured.

Question 8

This is a very rapidly moving field and it is quite likely that more information has been added to the website in the time between this activity being written and the time you are doing it. At the time of writing, the situation was as follows.

Evidence for the association takes the form of a point-like source towards the centre of a young supernova remnant. Apart from the Crab and Vela, examples now include the following supernova remnants: G54.1+0.3, B1509–58, G292.0+1.8, E0102–72.3 (also referred to as E0102–72 on the Chandra website), G21.5–0.9 3C58, G11.2–0.3, IC 443, PSR 0540–69, Cas A (possibly), G320.4–1.2, and the pulsar B1509–58 in supernova remnant G320.4–1.2.

Not all of these sources are pulsars – some do not show any pulses. The non-pulsing sources mentioned on these pages are: Cas A, IC 443, G21.5–0.9.

Resources

Chandra home page http://chandra.harvard.edu/

XMM-Newton home page http://sci.esa.int/xmm/

Website addresses for specific pages within these sites are given in the answers to various questions.

S282 Astronomy

Stellar populations − Part 1

Stellar motion in the solar neighbourhood

The Open University

Study time: 4 hours

Summary

In this extended spreadsheet activity you are supplied with data tables of observations of the radial velocities of several hundred stars within about 300 pc of the Sun. The activity is split into two parts. In this part of the activity you will analyse these data in order to measure the motions of stars in our region of the Galaxy. In the second part, another activity called *Population II stars in the solar neighbourhood*, you will consider evidence for different stellar populations.

You should have read to the end of Section 1.2.4 of *An Introduction to Galaxies and Cosmology* before starting this activity and you should understand the terms: Doppler effect, proper motion, Galactic coordinates, metallicity, Population I, and Population II. This activity builds on some of the ideas that were introduced in the activity *Stellar distance and motion*, and you may find it useful to review your work on that activity prior to starting this one, along with revising Section 3.2.1 of *An Introduction to the Sun and Stars*.

Learning outcomes

- Appreciate the characteristics of the various stellar populations in the Galaxy.
- Experience using and interpreting observational data (positions and velocities).
- Appreciate the dynamic state of the Galaxy and the motions of stars.
- Gain skills in using computational and analytical tools (in this instance, spreadsheets) to interpret astronomical data.

Introduction to the activity

During your study of the Milky Way you will have read about a number of characteristics of the Galaxy, such as the major structural components and their masses, and the sorts of objects associated with these components and the ways in which they are distributed through space. However, learning this collection of facts is only a small part of studying astronomy, or for that matter, science in general. An important part of understanding science involves understanding *how* the knowledge has been acquired. This is not a journey into history, but a journey into the scientific process.

Our current understanding of the Galaxy is built upon two inseparable pillars: observation and theory. Astronomical observations tell astronomers what the Galaxy looks like, but are influenced by the steps taken in making the observations. These steps may not provide a representative view of the Galaxy if factors influencing the observations have not been fully taken into account. Such a factor is absorption of light by dust, which is discussed in *An Introduction to Galaxies and Cosmology*. Astronomical theory, on the other hand, is built on a range of sciences including physics, mathematics, chemistry, geology/Earth sciences and biology that can be well studied on Earth. Often, however, Earth-based theory gives only a simplified treatment of complex astronomical phenomena, or does not deal with the same energy or size ranges found in astronomical settings. It is for these reasons that advances in astronomy require progress jointly in both theory and observations.

Observations are used to test and develop theory, and theory is used to help scrutinize and interpret observations. An important link between the two involves comparing observational data sets with models of the objects and/or phenomena studied. This often involves computing an idealized model of the data and comparing that with the real observations. This activity will take you through such a comparison, using a spreadsheet to handle the data manipulation.

Introduction to the science

You know already that most stars in the disc of the Milky Way move in orbits which are approximately circular. However, the motions are not perfectly circular, and there are slight differences in the motions of stars relative to one another. From our perspective as observers on the Earth, we can in theory measure two components of motion for each star:

1. the component along our line of sight to the star, which is called the radial velocity, and
2. the component across our line of sight to the star, which is its motion in the plane of the sky, called the transverse velocity.

Question 1

How might you measure the radial velocity of a star? (*Hint*: consider what you have learnt about the spectra of stars.)

What about the transverse velocity of a star; how would you measure that? Measuring a transverse velocity accurately is hard, because you need to calculate that velocity from two other measurements, both of which are difficult to make. The transverse velocity must be calculated from the measured distance of a star and its measured proper motion. Distances are often accurate to no better than a few per cent, and often they are a *lot* less accurate. The motions of stars across the sky, even over periods of a decade or more, are usually small compared with the apparent size of a stellar image, which is typically 1 arc second (usually abbreviated arcsec), so proper motions are also hard to measure accurately. This means that both the distance and proper motion of a star are often poorly known, and hence the star's transverse velocity is also poorly known. In contrast, it is quite straightforward to make measurements of the Doppler shift of a star. It can be performed to almost arbitrary precision, relatively easily to $\pm 1\ \mathrm{km\ s^{-1}}$, and to even finer precision, $\sim 5\ \mathrm{m\ s^{-1}}$, if extra effort is made.

How can an astronomer make reliable measurements of objects' velocities in the Galaxy, some at large distances from the Sun, if only one of the two components can be measured accurately? This may sound like a rhetorical question, but it isn't really. The answer is: by being clever.

The famous astronomer S. Chandrasekhar suggested in 1942 that the difficulty of measuring transverse velocities could be circumvented by measuring the radial velocities of stars in special directions. For example, if you observed stars in the direction on the sky directly opposite the Galactic centre, a direction that we call the Galactic anti-centre, the radial velocities of stars would correspond to their motions directly towards or away from the Galactic centre. The same applies to stars observed towards the Galactic centre.

From *An Introduction to Galaxies and Cosmology*, Section 1.2.4, you should recall that the Sun and other stars in the solar neighbourhood are orbiting the Galaxy clockwise when viewed from the north Galactic pole, moving more or less towards Galactic coordinates $l = 90°$ and $b = 0°$. If you measured the radial velocities of stars in this particular direction, then you would be measuring how much their orbital motion differs from that of the Sun. You could do the same by looking towards $l = 270°$ and $b = 0°$. Similarly, if you measure the radial velocities of stars by looking towards the north or south Galactic poles, then the radial velocities will reveal how fast stars are moving into and out of the Galactic disc in the vertical, or z, direction.

This technique allows astronomers to make accurate velocity measurements in these very important directions in the Galaxy, which are called 'cardinal directions'. The velocity components measured relative to the Sun in the three directions $(l, b) = (180°, 0°)$, $(90°, 0°)$ and $b = 90°$ are given the symbols U, V, and W respectively, and are positive for motions directed away from the Sun towards those directions.

Question 2

Why don't we need to specify the value of l for motion in the direction corresponding to the W velocity?

Question 3

Sketch a diagram of the Galaxy showing the location of the Galactic centre and the Sun, and the three cardinal directions described above. Label them as the Galactic anti-centre, the rotation direction and the north Galactic pole (NGP). Also label these according to the velocity components, showing the directions corresponding to positive velocities.

Why are these directions important? For instance, why not make measurements at $l = 30°$ and $b = 40°$, say? Well, the cardinal directions are important because they probe the symmetry axes of the Galaxy. The motion measured in the direction of the Galactic anti-centre indicates the motion in or out of the Galaxy's central gravitational potential; motion in the rotation direction tells you about the rotational speed and hence the angular momentum of the stars; motion towards the Galactic poles tells you how much energy the stars have to rise temporarily above the plane of the Galaxy.

A series of observations exploiting this technique was made by astronomers A. Sandage and G. Fouts. They measured the radial velocities of approximately 1300 stars in three groups located within 10° of the Galactic anti-centre, rotation

direction, and NGP. They published the radial velocities they measured in *The Astronomical Journal*, March 1987, Volume 93, page 592–609. We will use their data to examine the motions of stars in the solar neighbourhood.

Question 4

Why do you think Sandage and Fouts measured stars within 10° of the cardinal directions, rather than exactly in the cardinal directions?

The activity

Data tabulations

Three data files (u.dat, v.dat, and w.dat) have been used in this activity, containing observations made by Sandage and Fouts. The data files are publicly available, and were obtained via the website

ftp://cdsarc.u-strasbg.fr/pub/cats/III/145/

You can download the files yourself if you wish, as they are quite small, but we have already provided the data from these files in spreadsheets on the S282 DVD to save time (details follow in the next sections of this activity).

The first five lines of the file u.dat are shown in Figure 1. The first column gives the name of the star, e.g. SAO76948. Columns 2 and 3 give the coordinates of the star in equatorial coordinates, i.e. right ascension (RA) and declination (Dec). Right ascension can be measured in degrees from 0° to 360°, but it is more usual to divide the 360° into 24 hours, each hour into 60 minutes, and each minute in 60 seconds, and then to express the right ascension in h, min and s. That is the convention that has been adopted in u.dat, and SAO76948 has a right ascension of 05h 01min 47s. The declination is measured in degrees, arc minutes, and arc seconds, so the declination of SAO76948 is 29° 54′ 27″. Column 4 indicates the spectral type of the star, columns 5 and 6 give its brightness as *V* and *B* magnitudes, and the final column gives its radial velocity in km s^{-1}.

```
SAO76948   50147 295427 K0  7.8  9.2  -40.4
SAO76958   50302 294156 F0  8.0  8.5   19.3
SAO76983   50608 293200 G5  8.9  9.8    6.4
SAO76988   50632 290453 G0  8.6  9.7  -20.5
SAO76989   50633 294411 F8  6.6         6.0
```

Figure 1 The first five lines of the file u.dat.

The spreadsheets

The three data files have already been loaded into a spreadsheet called
`s282_gc11_start.sxc`

(the Excel version of the file is called `s282_gc11_start.xls`).

- Start the S282 Multimedia guide program and open the folder called 'Our Galaxy', then click on the icon for this activity (Stellar populations – Part 1).
- Press the Start button to access the folder on the DVD containing the StarOffice and Excel versions of the raw data file.
- Open the file you wish to use by double-clicking on it.
- As soon as it is open, save it with a new name to your hard disk, using the File | Save As... option. Make sure you note which folder of your hard disk you save it into, so you can easily find it again.

Making histograms of the radial velocities

The first task is to examine the range of radial velocities which have been measured for the stars. In the spreadsheet file, you will find three sheets named U, V and W, which correspond to the three input files. To select a different sheet, you click on the named tab at the bottom of the screen.

- Begin by clicking on the tab for sheet W.

You can examine the velocities by using the Page Down and Page Up keys to scan through the table, but this will give you only a very cursory view of what is in the table. A more scientific approach is to plot a histogram of the velocities. A histogram is a graph which has the desired independent variable (in our case the *W* velocity) along the horizontal axis, arranged into intervals, also called bins, of some chosen width. The vertical axis plots the number of data entries that fall within each bin. An example of a histogram common to students would be one giving the number of students who have achieved certain grades in an examination. The histogram would have exam scores along the horizontal axis, perhaps in bins of 10 marks, and the number of students achieving this score along the vertical axis. An example of such a diagram is given in Figure 2.

	A	B	C	D	E	F	G
1	**Student**	**Mark out of 100**		**Min**	**Max**	**mid-range**	**Frequency**
2	Anne	58		0	10	5	0
3	Bob	59		11	20	15	0
4	Cath	42		21	30	25	0
5	David	86		31	40	35	1
6	Erica	72		41	50	45	2
7	Felix	51		51	60	55	3
8	Gemma	82		61	70	65	2
9	Harry	39		71	80	75	3
10	Isabella	68		81	90	85	2
11	Joanne	75		91	100	95	0
12	Kevin	72			>100		0
13	Margaret	48					
14	Neil	63					

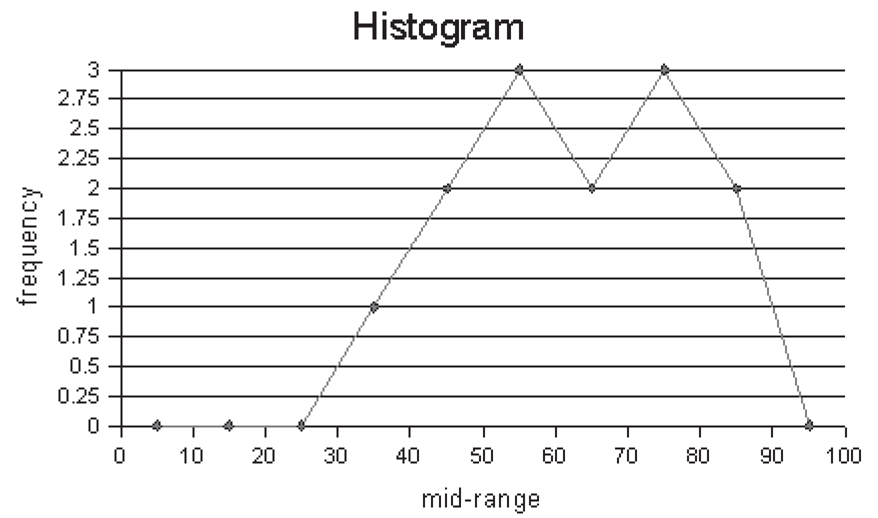

Figure 2 Example of a histogram and associated data tabulations showing the number of students scoring certain grades in an examination. The horizontal axis has been divided into bins 10 marks wide. (Histograms are usually, but not always, drawn as bar-graphs which have flat tops within each bin. Here, for convenience of plotting, we show them as single points at the mid values of each bin.)

In order to see what range of radial velocities is present in data file W, we should plot a histogram of the velocities in the same way we have for the examination scores in Figure 2. This is done as follows. (Note that a completed version of the spreadsheet (`s282_gc11_finish.sxc`) is provided on the DVD in the spreadsheets folder. If you have difficulties with your calculations, you may find it helpful to refer to the completed version to see how it has been done.)

First you need to discover what range of values will show up in the histogram. You could do this by scanning your eye through the file, but you can find out exactly by sorting the file according to the radial velocities, and then seeing what are the largest and smallest values.

- Click on any cell in the sheet and then type Ctrl-A to select (highlight) the entire table.

- Then sort the table according to the contents of column G, which is the radial velocity column. This is done by clicking on Data | Sort... | Sort by Column G | Ascending | OK. Note down the lowest and highest values in the radial velocity column (you will have to use Page Down to reach the bottom of the table), in this case –87.1 and 95.1 km s^{-1}.
- Finally, undo the sort so that you return to the original format of the file (Edit | Undo).

Next you need to decide how large each velocity bin should be in the histogram. Sandage and Fouts claim that each of their radial velocity measurements is accurate to about 4.7 km s^{-1}, so there is little to be gained by using velocity bins smaller than this. It helps to use nice round numbers, so intervals of 5 km s^{-1} would be appropriate, provided this gives more than just a few stars per velocity bin. If this produced too few stars per bin, we could increase the bin size to, say, 10 km s^{-1}.

Considering the data ranges found above, we will begin by using a velocity scale ranging from –100 to +100 km s^{-1}, binned over intervals of 5 km s^{-1}. To set up the histogram, we need to work out how many stars fall into each velocity bin. This count of stars per bin is called the 'frequency', and is easy to calculate with a spreadsheet. We then set out this information as a table from which we can generate plots.

- Construct a table in sheet W which indicates the lower and upper radial velocity (v_r) limit on each bin, and the midpoint velocity of each bin. It will be enough initially to enter these values for just the first two bins, those running from –100 km s^{-1} $\le v_r <$ –95 km s^{-1}, and –95 km s^{-1} $\le v_r <$ –90 km s^{-1}. You could begin by putting the heading 'lower limit' in cell M1 of sheet W, and then setting out the adjacent cells as in Figure 3.

M	N	O	P
lower limit	upper limit	midpoint	frequency
-100	-95	-97.5	
-95	-90	-92.5	

Figure 3 Layout of cells when beginning to set out a frequency table.

Once this is done, you can easily extend the table to the full velocity range as follows.

- Select (highlight) the first two rows of numerical values that you have just entered. To do this, depress and hold down the left-hand mouse button on the cell containing the value –100, and drag the cursor to the cell containing the number –92.5, then release the mouse button. You will see that at the lower right corner of the selected cells there is a small black square.
- Click and hold down the left-hand mouse button on this square, and then drag it vertically downwards, continuing to hold down the left-hand mouse button. You will see that a small counter appears; this indicates the numerical value that the first column will contain.
- Continue to drag the cursor down until this counter reaches the value 95, which is the lower limit of the last cell you need (the cell 95 km s^{-1} $\le v_r <$ 100 km s^{-1}). Then release the mouse button. You will see that this operation has extrapolated the bin ranges over the full range required.

The next step is to obtain the frequency values, i.e. the number of stars which fall into each radial velocity bin. This is done using a built-in function in the spreadsheet called 'frequency'. 'Frequency' calculates the number of data values that fall below the upper limit of each bin, but which equal or exceed the lower limit of the bin. It also calculates the number of data values that exceed the highest bin. (If you have set up the bins to cover the full data range then this number will be zero, but the spreadsheet computes it all the same.)

- Note down the first and last cells containing radial velocity values in sheet W; these should be cells G2 and G421.
- Note down the first and last cells that contain the upper boundaries of the velocity bins; these should be cells N2 and N41.
- Select the empty cells into which the frequency values should be put. In the column that we labelled frequency above, you should depress the left-hand mouse button in the cell just below the heading, and drag the cursor down to the right of the last entry in the midpoint column, 97.5, *and to the next cell down as well*, to allow the spreadsheet room to calculate the frequency of data values that exceed your last velocity bin. This process will cause you to highlight cells P2 to P42.
- Finally, with the cells still highlighted, begin to type in the formula to activate the frequency calculation. As this formula needs to know about two types of data, namely the individual radial velocities and the bin boundaries, it is a special kind of formula called a 'matrix' formula ('array' formula in Excel), and is entered slightly differently to formulae you have used before. With cells P2 to P42 highlighted, type the formula `=frequency(g2:g421;n2:n41)` but instead of just ending this with a normal carriage return (Enter), *use the three-key sequence* Ctrl-Shift-Enter (i.e. press Ctrl and hold it down, then press Shift and hold it down, and finally, whilst these two keys are held down, press Enter). You will then see that the highlighted cells will be filled with the number of stars whose radial velocities fall into each bin range.

It is now a simple exercise to plot the histogram of velocities.

- Select (highlight) the midpoint values for the bins, then hold down the Ctrl key and select (highlight) the frequency values, ignoring the final frequency cell (which should be a zero in any case) for which there is no corresponding midpoint value. *Check*: you should have highlighted the block of cells from O1 to P41.
- Clicking on Insert | Chart... will bring up the plot-building window, which takes you through four stages of constructing the plot. The second stage allows you to select the type of plot, where you should choose an XY chart, which is second from the left on the second line of choices. Also set Data series in Columns (if this is not already selected). The third stage allows you to Choose a variant; select Lines with symbols, which is the second option from the left. In completing the histogram, it is advisable to select Grid lines for both the X axis and Y axis. Give sensible axis labels as well, and a graph title. Once you have clicked on Create, the final graph will appear near the top of the sheet.
- You can move and/or resize the plot by clicking on its borders, and then dragging the boundaries with the cursor. Moving its upper-left corner to R6, and resizing its lower-right corner to Z27, is a convenient location and size.

- If you have a printer, you can print the graph and the associated frequency table. Highlight the block of cells from M1 to Z42, and then click on File | Print... . When the print dialogue window appears, chose Selection in the Print range.

You now have a histogram of the *W* velocity distribution. So, what does it mean? You will begin to interpret the data in the next section of this activity, but, just in case you are looking for a suitable point at which to take a break, this may be a good point at which to do so.

Interpreting the data

Now consider the histogram that you have plotted for the *W* velocities. It shows that the vast majority of stars have *W* velocities in the range –40 to +20 km s^{-1}. This suggests that the velocities that carry stars vertically above or below the disc are generally only a few tens of km s^{-1}. You might also note quite quickly that the distribution of velocities is not centred on zero, but is displaced to slightly negative velocities, centred around –10 km s^{-1}. Note that by definition, radial velocities are positive for stars that are receding and negative for stars that are approaching the observer. What does the non-zero average motion mean? It shows that stars viewed towards the North Galactic Pole are, on average, approaching the Sun. Why should this be the case? What is it about the Sun, or the position of the Sun in the Galaxy, which would make stars on average approach it? Consider this for a few minutes before proceeding.

In considering the question posed above, you might have proposed that the gravity of the Galactic disc is attracting more stars towards the disc than it allows to escape. If we were discussing gas clouds, you might indeed see this happening, but for stars there is a problem with such a proposition: stars almost never collide with anything substantial, so as they approach the disc with a certain speed, nothing absorbs their kinetic energy, and consequently they leave the disc on the opposite side with the same kinetic energy, and hence the same speed, that they approached it. That is, stars are not piling up near the centre of the disc. (In fact, the opposite occurs. Stars do interact with giant molecular clouds, but this tends to increase rather than decrease the heights to which stars travel, and hence causes a small net drift of stars away from the Galactic disc.)

The answer to the question, 'What is it about the Sun, or the position of the Sun in the Galaxy, which would make stars on average approach it?' is 'Nothing!' But if there is nothing special about the Sun, why do stars appear to be flowing towards it? The answer is that it is the Sun that is drifting relative to the neighbouring system of stars, rather than those stars drifting relative to us. The average motion of disc stars in the solar neighbourhood defines what is called the 'local standard of rest'.

You can make a precise measurement of the average *W* velocity of the sample observed towards the NGP above by calculating the average of their radial velocity values in the spreadsheet.

- If you click into an empty cell S1, and type the formula `=average(g2:g421)` then cell S1 will report the mean radial velocity, in this case –10.21 km s^{-1}.

When you perform a calculation of this sort, it is good practice to also insert a label saying what it is that you have calculated, and to give the appropriate physical units.

- Click into cell R1, and give the label average rv.
- Click into cell T1, and give the units km s^-1.

The negative sign on the radial velocity means that the mean radial velocity is directed towards the Sun. The finding that the mean speed of stars viewed in the direction of the NGP is about 10 km s^{-1} towards the Sun indicates that the Sun is rising up out of the disc, toward the NGP, at a speed of 10 km s^{-1}.

How accurately do we know this mean velocity? Obviously, adding one extra star to the sample would change the average slightly, so the average velocity depends on exactly which stars are included in our sample. In principle, we could measure so many stars that the effect of adding additional individual stars makes essentially no change to the average, but in reality it isn't feasible to do that. Consequently, we have to base our measurement of the mean velocity on just a sample of stars, in this case a few hundred of them. Fortunately, when the histogram of data values has a bell-shape, as the *W* velocity histogram does, there is a simple formula that gives an estimate of the uncertainty in the average of the data. The formula is used to calculate a quantity called 'the standard error in the mean', where the word 'mean' means 'average'. Sometimes this expression is abbreviated to just 'standard error'. The standard error in the mean depends on how spread out the data are and how many points have been measured. The spread of the data is measured by a quantity called the 'sample standard deviation', σ, which the spreadsheet will calculate. The standard error in the mean, *se*, is given by the sample standard deviation of the distribution divided by the square root of the number of points, *n*, in the sample:

$$se = \frac{\sigma}{\sqrt{n}}$$

To calculate the standard error in the mean, we therefore need to calculate the standard deviation and work out how many stars we have *W* velocities for.

- In cell S2, just below the calculation of the average, the sample standard deviation can be calculated by entering the formula `=stdev(g2:g421)`.
- In cell S3, the number of points can be calculated by using the formula `=count(g2:g421)`.
- In cell S4, the standard error can be calculated using the formula `=S2/SQRT(S3)`.

Cell S4 shows that the standard error in the mean *W* velocity is 1.06 km s^{-1}, so we conclude that the mean radial velocity of stars in the direction of the NGP is -10.2 ± 1.1 km s^{-1}.

What is the significance of this calculation for the *W* velocities? You have just measured the mean speed of the Sun in one of the three cardinal directions in the Galaxy, and have made an estimate of the uncertainty in that measurement.

In the next part of the activity, you will apply what you have learnt to the data measured in the two other cardinal directions to derive information on the *U* and *V* velocities as well. Before doing this, you should add appropriate labels and units to columns *R* and *T* respectively for the quantities you have just calculated. Now would be a good time to look back over what you have done so far in this activity, to clarify any uncertainties, and, if you wish, to take another break.

The three-dimensional space motion of the Sun

You now know enough about analysing the observational data in the spreadsheet to calculate the mean velocities for the other two cardinal directions as well, and hence to work out the full three-dimensional motion of the Sun. Return to the spreadsheet now, and calculate the mean motions for stars in the direction to the

Galactic anti-centre and the rotation direction. Although you do not need to view the histograms in order to calculate mean velocities, you will need to examine them later, so do plot them. When you have calculated the mean radial velocities and the uncertainties for the stars in each of the three cardinal directions, record the results in Table 1. When you have done so, answer the following question.

Table 1 Results of velocity measurements of stars in the three cardinal directions.

	U	***V***	***W***
Mean velocity/km s^{-1}			–10.2
Standard error in the mean/km s^{-1}			1.1

Question 5

What velocity components do you infer for the *motion of the Sun* in the Galaxy based solely on the mean radial velocities of stars from the observations by Sandage and Fouts? Be careful to explain in what direction the Sun is moving according to these calculations.

The total speed, υ_{LSR}, of the Sun relative to the local standard of rest can be calculated as

$$\upsilon_{\text{LSR}} = \sqrt{u^2 + \upsilon^2 + w^2}$$

where u, υ and w are the velocities of the Sun relative to the LSR calculated in Question 5. Doing this for the Sun's measured motion, (–3, 13, 10) km s^{-1}, indicates that its speed relative to the LSR is 17 km s^{-1}.

It is possible to use the inferred velocity of the Sun to calculate its orbit in the Galaxy. Such a calculation is beyond the scope of this activity, but shows that the Sun travels no more than about 100 pc from the midplane of the Galactic disc. That is, it will remain within the disc in the future, so it is a member of the disc population, Pop. I. It is not a Pop. II star that just happens to be passing through the disc.

Be sure to save all of your results from this activity, as you will need them for Part 2 this stellar populations activity, *Population II stars in the solar neighbourhood*, and then attempt the final two questions.

Question 6

What other evidence is there (from your studies, rather than from this activity) that the Sun is a Pop. I star?

Question7

To complete your work on this activity, write a brief summary of the observations you have used, and what you have shown about the stars in the solar neighbourhood.

Answers to questions

Question 1

The radial velocities of stars can be measured via the Doppler shift of their spectra, since the Doppler shift indicates motion along the line of sight to an object.

Question 2

Motion in the W direction is towards the north Galactic pole, and since all lines of longitude converge at the pole, it is enough to specify just its Galactic latitude, which is $b = 90°$.

Question 3

Your figure should resemble Figure 4 below.

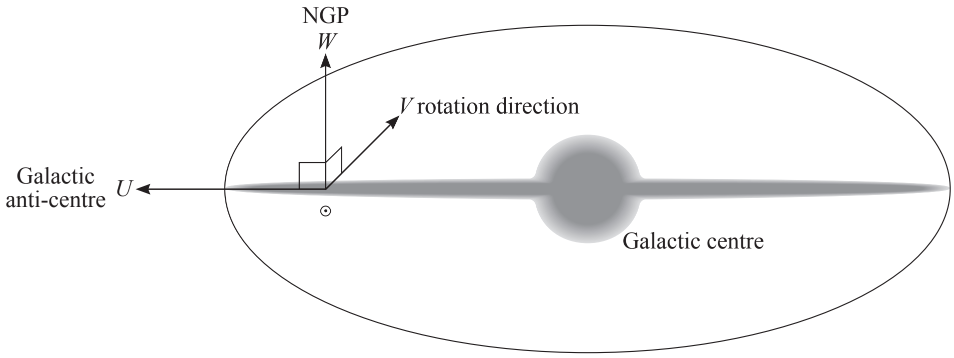

Figure 4 Schematic diagram of the Galaxy showing the cardinal directions corresponding to the velocity components U, V and W.

Question 4

Strictly, the cardinal directions are of infinitesimal size, so the probability of there being even one star exactly in any of these directions is negligible. This makes it necessary to measure stars within some angle of the cardinal directions. There is a compromise to be reached between going to a large enough angular separation to include a sufficiently large number of stars, yet not so far that the radial velocities no longer correspond closely enough to the desired velocity component U, V or W. Sandage and Fouts regarded the error made in going up to 10° away from the poles to be tolerable.

Question 5

As a result of your analysis you should have obtained values consistent with those given in Table 2.

Table 2 Results of velocity measurements in the three cardinal directions.

	U	V	W
Mean velocity/km s^{-1}	3.4	−13.2	−10.2
Standard error/km s^{-1}	1.5	1.4	1.1

The table shows that the uncertainties are in the range 1.1 to 1.5 km s^{-1}, so it is sensible to state the mean velocities to the nearest 1 km s^{-1}. (The mean value of some variable x is sometimes denoted $< x >$.) From the mean radial velocities of the observed stars, $(< U >, < V >, < W >) = (3, -13, -10)$ km s^{-1}, we infer that the Sun's (u, v, w) velocities relative to the local standard of rest are $(-3, 13, 10)$ km s^{-1}. That is, the Sun is moving towards the Galactic centre at 3 km s^{-1}, it is overtaking the motion of the local standard of rest by 13 km s^{-1} in the rotation direction, and moving towards the north Galactic pole at 10 km s^{-1}. These values compare very favourably with the generally accepted values, based on stars better representing the local standard of rest, $(-9, 12, 7)$ km s^{-1}.

Question 6

The Sun has a high metallicity, $Z \sim 0.02$, and its age is 4.5×10^9 yr, which are (respectively) higher and younger than are typical of Pop. II stars.

Question 7

We began with radial velocities for 420–450 stars in each of the three cardinal directions $(l, b) = (180°, 0°)$, $(90°, 0°)$ and $b = 90°$, located within about 300 pc of the Sun. By plotting velocity histograms, we found that most of these stars have velocities which differ from that of the Sun by no more than a few tens of km s^{-1}. By assuming that there is nothing special about the Sun or its location in the Galaxy, we have used the observed mean motions of the stars in the solar neighbourhood to infer the motion of the Sun relative to the local standard of rest. We find that the Sun is moving towards the Galactic centre at 3 km s^{-1}, is overtaking the motion of the local standard of rest by 13 km s^{-1} in the rotation direction, and moving towards the north Galactic pole at 10 km s^{-1}.

S282 Astronomy

Stellar populations – Part 2

Population II stars in the solar neighbourhood

Study time: 2 hours

Summary

In this extended spreadsheet activity you are supplied with data tables of observations of the radial velocities of several hundred stars within about 300 pc of the Sun. The activity is split into two parts. In the first part, *Stellar motion in the solar neighbourhood*, you measured the motion of stars in our region of the Galaxy. In this part of the activity you will consider evidence for different stellar populations and examine some of their characteristics.

You should have read to the end of Section 1.4 of *An Introduction to Galaxies and Cosmology* before starting this activity and completed Part 1, *Stellar motion in the solar neighbourhood.*

Learning outcomes

- Appreciate the characteristics of the various stellar populations in the Galaxy.
- Experience using and interpreting observational data (positions and velocities).
- Appreciate the dynamic state of the Galaxy and the motions of stars.
- Gain skills in using computational and analytical tools (in this instance, spreadsheets) to interpret astronomical data.

Introduction to the activity

You know already that most stars in the disc of the Milky Way move in orbits which are approximately circular. In the first part of this activity you assessed the radial velocities of stars in the three cardinal directions $(l, b) = (180°, 0°)$ $(90°, 0°)$ and $b = 90°$, given the symbols U, V, and W respectively.

Question 1

The part of the Galaxy in the direction $(l, b) = (0°, 0°)$ is the Galactic centre. What names are given to the three directions corresponding to $(l, b) = (180°, 0°)$, $(90°, 0°)$ and $b = 90°$?

In Part 1 of this activity you plotted histograms for each of the three velocities, and calculated the mean velocity, the standard deviation, and the standard error in the mean. Although we did not dwell on it at the time, the standard deviation is a measure of the spread of the data points about the mean.

Question 2

What is the standard deviation σ that was measured for the radial velocities in each of the three cardinal directions? (Go back to your calculations in Part 1 to answer this.)

The activity

Modelling the velocity distributions

It was noted in Part 1 of this extended activity that the velocity distributions were roughly bell-shaped. We will now put that claim to a more rigorous test by computing the shape of a special bell-shaped curve that is called a 'normal' or 'Gaussian' distribution. Many natural processes give can rise to a distribution of a variable that has this shape. The approach we'll take is to compare our observed distributions of components of velocity with Gaussian/normal curves that have the same mean and standard deviation as each of our three samples.

The general expression for a normal distribution $f(x)$ of some independent variable x is the nasty-looking equation (which you shouldn't expect to remember)

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(-(x-\bar{x})^2/2\sigma^2\right)}$$

As an example, we let x represent the lengths (in mm) of objects which are all similar to one another. A plot of this function is shown in Figure 1, where the mean $\bar{x}$ is 100 mm and the standard deviation σ is 10 mm. The normal distribution is a symmetrical bell-shaped curve that reaches a maximum at the mean (=average) of the distribution, i.e. where $x = \bar{x}$. Where $x = \bar{x} + 2\sigma$ and $x = \bar{x} - 2\sigma$, the function has diminished considerably. The curve indicates that if you had a sample of objects whose lengths differed from one another according to this normal distribution, then most would have lengths between about 90 and 110 mm, while a few would be shorter than 80 mm or longer than 120 mm. Also, the total area under this curve is 1.0 (in dimensionless units).

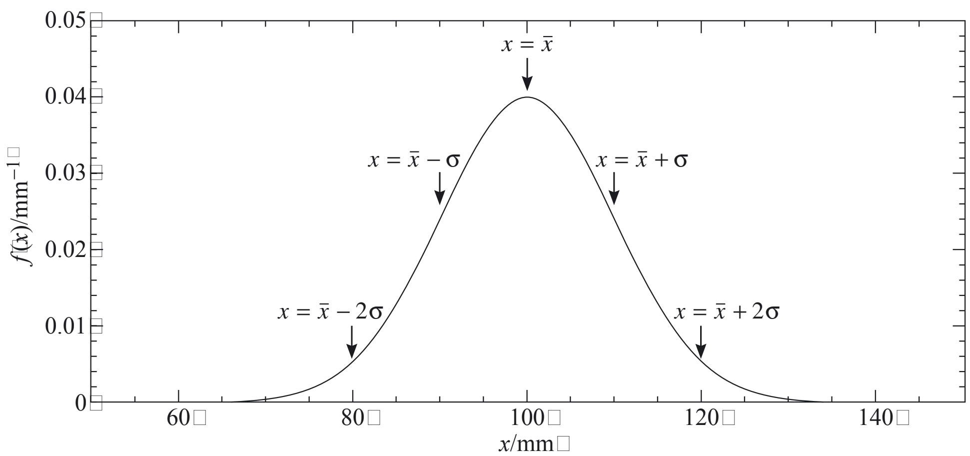

Figure 1 Normal distribution for $\bar{x}$ = 100 mm and σ=10 mm.

Begin by opening the spreadsheet that you completed in Part 1 of the activity. If you were unable to finish that activity, you might want to obtain the version of the final spreadsheet that has been supplied on the DVD.

Our aim is to plot a normal-distribution curve that can be compared with the observational data. To do this, we have to scale up the theoretical one, because the area under each of the three histograms calculated in Part 1 is more than 1.0. The area under a histogram of observational data is given by the number of stars in the data set, n, multiplied by the x-axis bin width, expressed in x-axis units. That is, since the x-axis bin width is 5 km s^{-1}, we multiply n by 5. That is, we need to rescale (multiply) the general formula for the normal distribution by $5n$.

Question 3

How many stars, n, are there in each of the three data sets?

The normal distribution can be readily computed for the data in the finished spreadsheet of Part 1 as follows.

- Begin with sheet U for the U-velocity, and put a heading 'normal' in cell Q1.
- Click in the next cell down, Q2, and prepare to type the formula for the scaled-up normal distribution.

What should that formula be? It will be easier if we work this out in pieces. From the expression for the normal distribution, we know we need an exponential term, $e^{\left(-(x-\bar{x})^2/2\sigma^2\right)}$. The exponential function is calculated within the spreadsheet using the built-in function `EXP( )`, and values are squared by following them with a carat symbol and the power two, `^2`. The velocity at the midpoint of each bin of the histogram is in column O, the mean velocity of the sample is in cell S1, and the standard deviation is in cell S2, so the exponential part of the formula can be typed

```
=EXP(0-(O2-$S$1)^2/(2*$S$2^2))
```

Note that dollar signs (`$`) have been used to freeze the cell references S1 and S2, so that the formula will pick up the entries for $\bar{x}$ and σ correctly when it is copied to other cells.

The other part of the equation for the normal distribution is the factor $\dfrac{1}{\sqrt{2\pi}\,\sigma}$.

The spreadsheet stores the value of π in the function `PI( )`, and the square root is calculated with the function `SQRT( )`, so we precede the exponential term by the factor

```
(1/(SQRT(2*PI())*$S$2))
```

Finally, we recall that we have to rescale the theoretical formula by a factor $5n$. Since the number of stars n is recorded in cell S3, we can enter the full formula in cell Q2 as

```
=5*$S$3*(1/(SQRT(2*PI())*$S$2))*EXP(0-(O2-
$S$1)^2/(2*$S$2^2))
```

- Having checked that this makes sense, copy (drag) the formula down to fill the column that is two cells to the right of each of the midpoint values, until you reach the cell on the same line as the last of the midpoint entries. This calculates the value of the rescaled normal distribution at each midpoint.

It is then a simple task to insert a new chart which plots both the histogram of stellar radial velocities and our approximate model of them, the normal distribution.

- Select (highlight) the three columns headed 'midpoint', 'frequency' and 'normal', and then clicking on Insert | Chart... to set up the new plot, following much the same process as in Part 1.
- Copy and paste the formula into each of the remaining sheets V and W, and produce new plots for all three velocities.

Here you see one of the beauties of working with spreadsheets; once you set up a formula, you can copy it not only to cells within the same sheet, but also to cells on other sheets which have the same layout (as ours do). If you wish to take a break, you may find this a convenient point at which to do so.

Interpreting the comparison

Now consider the histograms that you have plotted. Do you think these model distributions provide good fits to the observational data? (It would be possible to perform various statistical tests that would quantify how good the fits are, but that is unnecessary for the current purposes.) Consider each of the velocity components U, V, and W in turn, and reach an opinion about each one before proceeding.

The histograms for the U and W velocities look to be reasonable fits to the data, but it is clear that the V-velocity comparison is not very good. The modelled normal distribution is much too broad compared to the data, and could not be said to be a good fit. Assuming you have set up the equations correctly, and assuming the spreadsheet can handle the arithmetic correctly, what has gone wrong?

The fact that the calculated V-velocity distribution is too broad, and that the calculation of this distribution uses the value of the standard deviation σ_V that we calculated in Part 1, suggests that that value is not terribly useful. This is not to say it is wrong, but rather that it does not give a good representation of the data.

What is the difference between these two statements? The standard deviation indicates the spread of the data, that is for certain, but just because we can calculate a standard deviation does not mean the data conform to the normal distribution. The mismatch between the observations and the model curves for the V velocity is in fact telling us that the distribution is not normal. Your job as a scientist is to work out why, especially since the observed U and W velocities do give a reasonable match to the normal curve.

Inspection of the V-velocity histogram shows that even the calculated normal distribution, which we can see is too broad, is close to zero for $V < -100$ km s^{-1}. This suggests that there is something odd about the stars with velocities more negative than this. What might be wrong with these stars? A careful scientist might first enquire whether their radial velocity measurements were reliable. Once that is verified, the next step might be to ask whether these stars' velocities are meaningful. If, for example, these stars were members of binary systems so they were in orbit about a companion star, then their velocities would be telling us about the dynamics of their binary system, not about their Galactic motions. That is, the velocities would be accurate but inappropriate for our study. If, however, the stars are confirmed to be single stars, then we are left with a third conclusion, that their velocities are genuinely very different from those of other stars in the solar neighbourhood, and they are telling us about a system of stars other than the system which constitutes the local standard of rest. In fact, the last of these is the relevant conclusion in this case, and the five stars with $V < -100$ km s^{-1} are not part of the system of stars defining the local standard of rest. That is, they are not rotating around the Galaxy at roughly 220 km s^{-1} like most of the disc stars.

Question 4

If the stars with $V < -100$ km s^{-1} are not part of the system of stars defining the local standard of rest, what system might they be part of?

If the stars with large negative V velocities are correctly identified as not belonging to Pop. I, then we can use their properties to make rough estimates of the parameters of the population to which they do belong. A sort of the radial velocities (highlight column G and then click Data | Sort... | Ascending shows that the V values for these five stars are –314.3, –262.8, –166.2, –143.5, and –113.3 km s^{-1}. (Don't forget to do Edit | Undo sort to revert to the original file order.) If they belong to Pop. II, then we can use these values to estimate the rotation of Pop. II stars about the Galactic centre. This is the topic of Question 5.

Question 5

(a) By reusing the techniques you have used earlier in this activity to calculate the mean and standard deviation of a set of data, calculate the mean and standard error of the mean for the V velocities of the five Pop. II stars. (You may find it convenient to copy the V velocities for these five stars to a small table to the side of the existing tabulations in sheet V, and to make the calculation there. If you get stuck, look back at Part 1, in the section 'Interpreting the data', where means and standard deviations were first calculated. If you get *really* stuck, you can look into the solution spreadsheet for this activity which is supplied on DVD.)

(b) Assuming the Sun is overtaking the local standard of rest by 12 km s^{-1}, by what speed do Pop. II stars fall behind the LSR?

(c) Measurements of other astronomical objects suggest that the LSR moves around the Galactic centre at 220 km s^{-1}. Based on your measurements, at what speed do Pop. II stars orbit the Galactic centre (on average)? What does your result indicate?

(d) Use the data to estimate the approximate ratio of the number of Pop. II stars to the number of Pop. I stars in the solar neighbourhood.

Concluding remarks

Your analysis of the velocity distributions of the observed stars has led you to discern that a small proportion of the stars in the solar neighbourhood, about 1% of them, belong not to the disc population, Pop. I, but to the halo population, Pop. II. Furthermore, you have been able to estimate the mean rotation velocity of Pop. II stars as indistinguishable from zero, at 32 ± 38 km s^{-1}. This is in contrast to the rapid rotation of stars in the Galactic disc, which travel at about 220 km s^{-1}. To finish your analysis of the differences between the motions of Pop. I and Pop. II stars, consider the following question.

Question 6

(a) Comment on the standard deviation of the V velocity for the five halo stars compared to the overestimated value for disc stars that we calculated earlier.

(b) How might Pop. II stars be recognized by their velocities if they are observed not via radial velocities in the cardinal directions, but by transverse velocities in arbitrary directions?

Question 7

To complete your work on both parts of the extended activities on stellar populations, write a brief summary (approximately 250 words) of the observations you have used, and what you have shown about the stars and the stellar populations in the solar neighbourhood.

Answers to questions

Question 1

The directions (l, b) = (180°, 0°) (90°, 0°) and b = 90° are, respectively, toward the Galactic anti-centre, Galactic rotation direction, and North Galactic Pole. (Refer back to Part 1 of the activity.)

Question 2

$\sigma_U = 31\ \text{km s}^{-1}$, $\sigma_V = 29\ \text{km s}^{-1}$, $\sigma_W = 22\ \text{km s}^{-1}$.

Question 3

$n_U = 430$, $n_V = 445$, and $n_W = 420$.

Question 4

The stars with $V < -100\ \text{km s}^{-1}$ are being overtaken by the Sun by more than $100\ \text{km s}^{-1}$. One might reasonably suspect that they are really members of Population II rather than Population I.

Question 5

(a) For the five possible Pop. II stars, calculations of the mean velocity and its standard error give $\langle V \rangle = -200 \pm 38\ \text{km s}^{-1}$ (relative to the Sun).

(b) If the LSR is moving $12\ \text{km s}^{-1}$ slower than the Sun, then the Pop. II stars are rotating $200(\pm 38) - 12\ \text{km s}^{-1}$ slower than the LSR, i.e. $188 \pm 38\ \text{km s}^{-1}$ slower than the LSR.

(c) If the LSR is rotating about the Galaxy at $220\ \text{km s}^{-1}$, then on average Pop. II stars must be rotating at $220 - 188(\pm 38)\ \text{km s}^{-1} = 32 \pm 38\ \text{km s}^{-1}$. This indicates that Pop. II stars have almost no net rotation about the Galaxy.

(d) If 5 out of the 445 stars in the Galactic rotation direction belong to Pop. II, then this suggests the fraction of Pop. II to Pop. I stars is $5/(445 - 5) = 0.01$. It is clear that Pop. II stars are quite rare.

Question 6

(a) The standard deviation of the V velocity for Pop. II stars is 85 km s^{-1}, which is considerably larger than even the overestimated value of 29 km s^{-1} that we derived earlier for the disc stars. This shows that Pop. II stars have much larger random motions than Pop. I stars.

(b) Because of the larger random motions of Pop. II stars, they could be observed as stars having apparently high velocities compared to most other disc stars even away from the cardinal directions. These are the so-called 'high-velocity' stars.

Question 7

We began in Part 1 with radial velocities for 420–450 stars in each of the three cardinal directions $(l, b) = (180°, 0°)$, $(90°, 0°)$, and $b = 90°$, located within about 300 pc of the Sun. By plotting velocity histograms, we found that most of these stars belong to the Galactic disc (Pop. I) and have velocities which differ from that of the Sun by no more than a few tens of km s^{-1}, with standard deviations in the range 22–31 km s^{-1}. We used the observed mean motions of the stars to infer the motion of the Sun relative to the local standard of rest. We found that the Sun is moving towards the Galactic centre at 3 km s^{-1}, is overtaking the motion of the local standard of rest by 13 km s^{-1} in the rotation direction, and moving towards the north Galactic pole at 10 km s^{-1}. The V-velocity distributions showed that about 1% of the stars in the solar neighbourhood belong not to the disc population, Pop. I, but to the halo population, Pop. II. The mean rotation velocity of Pop. II stars was found to be indistinguishable from zero, at 32 ± 38 km s^{-1}, in contrast to the rapid rotation of stars in the Galactic disc, which travel at about 220 km s^{-1}. The standard deviation of the V velocity for Pop. II stars is 85 km s^{-1}, which is considerably larger than even the value for the disc stars. Because of the larger random motions of Pop. II stars, they are sometimes recognized in the solar neighbourhood as 'high-velocity' stars.

S282 Astronomy

Galaxy classification

Study time: 30 minutes

Summary

In this activity you will classify galaxies into the various Hubble classes described in Chapter 2 of *An Introduction to Galaxies and Cosmology*. This activity uses a set of images of nearby galaxies which is provided on the Image Archive on the S282 DVD.

You will also need a transparent ruler marked in millimetres.

You should have read to the end of Section 2.2 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- To be able to classify galaxies according to the modified Hubble scheme.
- To appreciate that there is some degree of subjectivity in the classification process.

The activity

You will use the Hubble classification scheme described in Section 2.2 of *An Introduction to Galaxies and Cosmology* to classify galaxies by their morphology, using images from major observatories around the world provided in the Image Archive.

- Start the S282 Multimedia guide, and open the folder called 'Galaxies', then click on the icon for Galaxy classification.
- Press the Start button to launch the Image Archive at the required set of images.
- Alternatively, launch the Image Archive by clicking the Image Archive, button in the Multimedia guide and then find the Galaxy Classification set which is located within the Galaxies section of the archive.

You should now see 24 thumbnail negative images of galaxies. As usual, you can move the cursor across the images to see what they are – the NGC (New General Catalogue) number will appear – and you can obtain a larger image of each by clicking on the thumbnail. The first thing you will do is classify the galaxies roughly into the Hubble classes.

■ What are the four major Hubble classes?

❑ Elliptical, lenticular, spiral and irregular; lenticular and spiral are also divided into barred and unbarred subclasses.

You will then determine the Hubble types of a selected number of galaxies.

Question 1

First, recall the Hubble types and note them down for each class in Table 1, giving a brief description of each class and its types, as in the example of the lenticular class shown.

Table 1 Descriptions of the Hubble classes and types (for use with Question 1).

Hubble class		Hubble types	Characteristics determining the class and type
Elliptical			
Lenticular	Unbarred Barred	S0 SB0	Lens-shaped with disc and nuclear bulge but no spiral arms. The bulge of an S0 galaxy is a spheroid, but that of an SB0 is a bar.
Spiral	Unbarred Barred		
Irregular			

Table 2 corresponds to the grid of 24 thumbnail images on the Image Archive. Fill it in with the Hubble classes and types as you work through the activity (or make a separate note of your classifications).

The symbols [X] and * indicate the order in which we suggest you classify the galaxies: we will work through those marked with [X] as examples to get you going, and then leave you to work through the classification of those marked with * with a discussion of how each one is classified at the end of the activity. The rest we leave as an optional exercise (though we will give you the answers!)

Table 2 The grid of thumbnail images in the order in which they are displayed in the Image Archive.

NGC 7814	NGC 0024	NGC 0134	NGC 0147	NGC 0148*	NGC 0150*
NGC 0157	NGC 0185	NGC 0205*	NGC 0210[X]	NGC 0221	NGC 0255
NGC 0278	NGC 0488*	NGC 0514	NGC 0524	NGC 0615	NGC 0636*
NGC 0681*	NGC 0720[X]	NGC 1052	NGC 1073*	NGC 1156*	NGC 1172

Example 1: NGC 0210

Well, I hope you agree that this is a spiral galaxy! But let's be a little more systematic: to determine the Hubble class ask yourself:

- ■ Is there any overall regularity or symmetry?
- ❑ Yes. The galaxy appears to be an ellipse, and would look the same if you turned it upside down. So it can't be an irregular.

- ■ Is there any internal structure?
- ❑ Yes. There is a clearly defined bright nuclear bulge in the centre, surrounded by two impressive spiral arms. (Remember that the images are negative so the bright features look dark.) So it can't be an elliptical.
- ■ Are there any spiral arms?
- ❑ As above, yes. So it's not lenticular, but spiral.

This means that the true shape is a circular disc with a nuclear bulge. The disc appears in the image to be an ellipse rather than a circle so we're not looking face on to it but at an angle; this angle of inclination is a little over 45°. The largish bright object at the 4 o'clock' position is a foreground star in our Galaxy, but the spiral arms contain many small bright HII regions, probably of star formation. Furthermore the spiral arms are branched towards their ends. There is no sign of a central bar, so this galaxy is an S, not an SB.

Determining the Hubble type of a spiral is quite hard and also quite subjective – if you look them up in different sources you don't always get the same answer!

(I find that one way to remember the order of Sa – Sc is that the arms of Sa are tightly wound like the symbol '@', whereas the arms of Sc are loosely wound like C.) As well as looking at how tightly the arms are wound, you need to look at the relative size of the nuclear bulge. Here, the arms are quite definitely wrapped around the fairly large nuclear bulge, so it's unlikely to be Sc, but at the same time they are reasonably open – you can see space between the arms and bulge, so it's unlikely to be Sa.

Hence for NGC 0210 we assign the Hubble type Sb.

Example 2: NGC 0720

This looks quite different from the previous example. Again let's be systematic in determining the Hubble class:

- ■ Is there any overall regularity or symmetry?
- ❑ Yes. The galaxy appears to be a smooth ellipse, so it can't be an irregular.
- ■ Is there any internal structure?
- ❑ No, not really. There are no distinct features, but the brightness is concentrated in the centre and decreases steadily towards the edge of the galaxy. Indeed as it fades out, you can't really say where the edge is. So it is an elliptical galaxy, E.
- ■ Are there any spiral arms?
- ❑ No, and you wouldn't expect them in an elliptical galaxy.

With practice you can judge the Hubble type just by looking at the shape of the ellipse, but to start with you will be better off measuring the axes and calculating the flattening factor.

Recall that the type of an elliptical galaxy is defined as the nearest whole number to $10 \times (a - b)/a$, where $(a - b)/a$ is the flattening factor, calculated from the semimajor axis a and the semiminor axis b of the visible ellipse (Section 2.2.1). You can measure these directly on the images; since you are dividing one length by another, the result is a dimensionless ratio (it has no unit) and so it doesn't matter that you are measuring the image rather than the real thing (which would be a little difficult!).

In fact, you can measure the major and minor axes – again, since you are calculating a ratio, it doesn't make any difference whether you measure $2a$ and $2b$ instead of a and b, you will get

$$\frac{2a-2b}{2a}=\frac{2(a-b)}{2a}=\frac{a-b}{a}$$

as required.

The tricky bit is deciding just where to measure the axes. As we have said, it's hard to say where the edge of the galaxy is. So you have to imagine a contour line of equal surface brightness around the centre region – an *isophote* – and measure the axes of that.

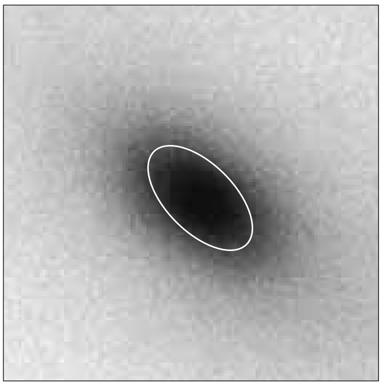

Figure 1 An isophote superimposed on NGC 0720.

You could try measuring directly from the screen, but it is better to print the image, and then sketch in an isophote and measure this. (Don't worry about the black surround of the image, the surround prints as white.)

I got the major axis, $2a = 35$ mm, minor axis, $2b = 18$ mm, so the flattening factor

$$\frac{a-b}{a}=\frac{(35-18)\text{ mm}}{35\text{ mm}}=0.49$$

(Your choice of an isophote is likely to differ from mine, and so your measurements may be quite different, but the final flattening factor should be similar.)

To get the Hubble type, we have to multiply the flattening factor by 10 and round to the nearest whole number: $0.49 \times 10 = 4.9 \sim 5$.

So for NGC 0720 we assign the Hubble type E5.

Don't be disappointed if your Hubble types for these two examples are slightly different from those given here. I'm sure you will have realized by now that judging just where to measure the axes of an elliptical is just as difficult as deciding how tightly wound the arms of a spiral are, and how large is its nuclear bulge. Even the experts sometimes disagree!

Question 2

Now have a go at classifying a few more galaxies yourself. Try the following eight, which give a good spread of types (these are the ones marked with * in Table 2):

NGC 0148, NGC 0150, NGC 0205, NGC 0488, NGC 0636, NGC 0681, NGC 1073, NGC 1156.

Have a look at the answers if you get stuck.

Question 3 (optional)

Finally, you might like to try classifying the remaining 14 galaxies in the galaxy classification section. The accepted classes and types are shown in Table 4 in the answers.

Answers to questions

Question 1

The answer is given in Table 3.

Table 3 Descriptions of the Hubble classes and types. (The completed Table 1.)

Hubble class		Hubble types	Characteristics determining the class and type
Elliptical		E0 – E7	Overall elliptical outline, featureless appearance, light output concentrated in centre, decreasing as you move outwards. The number gives 10 × the flattening factor – E0 appears circular, E7 is the most flattened.
Lenticular	Unbarred Barred	S0 SB0	Lens-shaped with disc and nuclear bulge but no spiral arms. The bulge of an S0 galaxy is a spheroid, but that of an SB0 is a bar.
Spiral	Unbarred Barred	Sa, Sb, Sc SBa, SBb, SBc	Circular disc (though may appear elliptical) with spiral arms and nuclear bulge. SB types have central bar as above. The letters a, b, c show how tightly wound the spiral arms are and how big the nuclear bulge is – Sa, SBa are tightly wound with a large bulge, Sb, SBb less so and Sc, SBc loosely wound with a small bulge.
Irregular		Irr	No symmetry or regularity.

Question 2

The classifications are discussed below.

NGC 0148

- ■ Is there any overall regularity or symmetry?
- ❑ Yes. The galaxy isn't quite an ellipse but it's certainly symmetrical, so it can't be an irregular.
- ■ Is there any internal structure?
- ❑ Yes. There is a distinct nuclear bulge, and a slight indication of a darkening between the bulge and the ends of the major axis. (Remember the image is a negative, so a darkening appears lighter). This is consistent with a disc with a bright outer ring; the darkening could be due to less bright matter or obscuring dust. The angle of inclination is so great (nearly 80°) that the bulge extends above and below the disc, giving it a not quite elliptical shape. So it is not an elliptical.
- ■ Are there any spiral arms?
- ❑ No, we can see no evidence of actual spiral arms. So it is not a spiral but a lenticular.

There is no evidence either of a bar in the bulge, so this is an unbarred lenticular S0 galaxy.

NGC 0150

- ■ Is there any overall regularity or symmetry?
- ❑ Well, it's not perfectly symmetrical, but there is a sort of regularity about it, so we'll leave the classification 'irregular' aside for the moment.
- ■ Is there any internal structure?
- ❑ Yes. There is evidently a disc of two or three spiral arms, quite tightly wound, but one of the arms spoils the symmetry, possibly rising out of the plane of the disc. There also seems to be a small bright nuclear bulge. So it is some kind of a spiral galaxy.

Despite the small nuclear bulge, which would tend to indicate Sc, the arms are really too tightly wound for this classification, so I would classify it as Sbc. Some (but not all) observers claim to detect a bar, and it is sometimes classified SBb. The strange asymmetrical rising third arm also qualifies it as a *peculiar* galaxy, so the full Hubble type I would give is Sbcp.

NGC 0205

- ■ Is there any overall regularity or symmetry?
- ❑ Yes. The image of the galaxy has an elliptical shape. So it's not an irregular.
- ■ Is there any internal structure?
- ❑ Not in the sense of a nuclear bulge or spiral arms. It looks like a typical elliptical in the sense of a generally bland appearance with the brightness concentrated in the centre and decreasing steadily towards the edge. But if you look closely you will see that there are a couple of darkened (i.e. pale-looking in the negative) patches on the right-hand side. These are dust patches, and dust, like gas, is rare in elliptical galaxies.
- ■ Are there any spiral arms?
- ❑ No. So it isn't a spiral.

The dust patches, together with other unusual features for ellipticals such as bright young stars which aren't really visible in this image, have led many observers to classify this as a lenticular, S0. But other observers stick with the perhaps more obvious classification as an elliptical. With a major axis of around 40 mm and a minor axis of 21 mm, the flattening factor is 19/40 = 0.48, so the Hubble type is E5, or E5p to take account of the peculiar feature of the dust patches.

This galaxy is M110, one of the dwarf companions of M31, the Andromeda Galaxy.

NGC 0488

I hope you agree that this is a spiral. The nuclear bulge is quite large and the thin multiple arms contain many small lumpy bright patches, indicating star formation. The arms are quite tightly wound, giving it the Hubble type Sab.

NGC 0636

I hope you agree also that this is an elliptical. It is not quite circular; the 10 o'clock – 4 o'clock axis is slightly longer than the 1 o'clock – 7 o'clock axis, giving a Hubble type of E1. What looks like a small satellite galaxy at 7 o'clock is actually an offset secondary image of the small bright nucleus of NGC 0636 itself.

NGC 0681

This is a bit trickier. Let's go back to the systematic approach:

- ■ Is there any overall regularity or symmetry?
- ❑ Yes. The two sides are more or less mirror images of each other.
- ■ Is there any internal structure?
- ❑ Yes. There is a bright extended nuclear bulge with a darkened thick dust lane obscuring the bottom half of it. This is consistent with a disc seen almost edge on.
- ■ Are there any spiral arms?
- ❑ Difficult to say, because of the angle and the obscuring dust, but at the sides of the nuclear bulge it looks as though there may be spiral arms in the disc.

The relative sizes of the nuclear bulge and the disc, and the general impression of not too tightly wound arms lead to a classification as Sb or Sab.

NGC 1073

Once again, this is clearly a rather beautiful spiral, seen almost face on. The nuclear bulge is small and distinctly barred, and the arms are open and branched. Hubble type SBc.

NGC 1156

- ■ Is there any overall regularity or symmetry?
- ❑ Not really. It is vaguely peanut shaped, with bright lumpy star forming regions. The star formation indicates that it is not an elliptical. Some observers claim to see it as a bar, others claim to see rudimentary spiral structure. But the general classification is as an irregular – Irr.

Question 3

The answer is given in Table 4.

Table 4 The completed grid of galaxy classifications.

NGC 7814 spiral edge-on possibly Sab	NGC 0024 spiral Sc	NGC 0134 spiral Sbc	NGC 0147 elliptical E5	NGC 0148* lenticular S0	NGC 0150* spiral/pec Sbcp
NGC 0157 spiral Sc	NGC 0185 elliptical E3	NGC 0205* elliptical/ lenticular E5p/S0	NGC 0210[X] spiral Sb	NGC 0221 elliptical M32 E2	NGC 0255 barred spiral SBc
NGC 0278 spiral Sbc	NGC 0488* spiral Sab	NGC 0514 spiral Sc	NGC 0524 lenticular S0	NGC 0615 spiral Sb	NGC 0636* elliptical E1
NGC 0681* spiral Sab/Sb	NGC 0720[X] elliptical E5	NGC 1052 elliptical E4	NGC 1073* barred spiral SBc	NGC 1156* irregular Irr	NGC 1172 elliptical/ lenticular E1/S0

Acknowledgement

Figure 1 From *The Carnegie Atlas of Galaxies*, Volume I, A. Sandage and J. Bedke, 1994, Carnegie Institution of Washington – obtained from NASA Extragalactic Database.

S282 Astronomy

Mapping the Milky Way

Study time: 45 minutes

Summary

This activity relates to a video sequence in which you will see how observations made at various wavelengths are used to by astronomers to gain insights into the structure of our Galaxy.

You can watch this video sequence at any time while studying Chapter 1 of *An Introduction to Galaxies and Cosmology*, but you may find it to be of more benefit during the second half of the chapter.

Learning outcomes

- Understand the structure of the Milky Way and how it has been discerned.
- Appreciate some of the key developments in our knowledge of the structure of the Milky Way.

The activity

This activity follows the historical development of our growing knowledge of the Milky Way, a mapping made all the more difficult because we are deeply embedded within the Milky Way, and see it from a single viewpoint.

- Start the S282 Multimedia guide and then click on Mapping the Milky Way under the 'Our Galaxy' folder in the left-hand panel.
- Press the Start button to run the video sequence.

After you have watched the video sequence, read the summary provided in the 'Notes' below.

Notes

You saw that in the 1920s the astronomer Edwin Hubble proved that many of the nebulae catalogued in the previous 150 years were huge assemblages of stars lying beyond our Galaxy – the Milky Way – rather than within it. Therefore, he had shown that the Milky Way is not the only galaxy, but that there are very many others. These other galaxies come in a variety of shapes and sizes, but which shape, if any, corresponds to the shape of our own Galaxy? Is it spiral, elliptical, irregular, or something else?

The development of radio astronomy was crucial in answering this question. This is because, at visible wavelengths, much of the Galaxy is obscured from view by interstellar dust. The importance of radio astronomy became apparent when the astronomer Hendrik van de Hulst predicted the existence of a spectral line that would be little attenuated by dust. This line is at the radio wavelength of 21 cm, and should be emitted by the interstellar atomic hydrogen that was thought to be present throughout the Galaxy. However, it was not until 1951 that developments in radiotelescopes gave sensitivities sufficient for the line to be detected.

Later studies of the 21 cm line, continuing today, have shown that the velocities of interstellar hydrogen clouds are too high to be explained by the gravitational field of the matter that can be seen in the form of stars and interstellar clouds. This is one indication that the Galaxy contains large quantities of dark matter, matter that so far is detectable only by its gravitational effect. This is an active area of enquiry, with largely unsolved problems regarding the nature and distribution of dark matter.

You then saw that infrared astronomy too has been of great importance in mapping the Milky Way, because, like radiowaves, infrared waves are little affected by dust. Infrared measurements are best made from above the Earth's atmosphere, and have revealed stars hidden by dust at visible wavelengths, and have improved our view right to the centre of the Galaxy.

A key feature of a map is its scale, and the distance scale in the Milky Way, for large distances, depends largely on using variable stars as standard candles. To calibrate the standard candle we need to measure the distance to some nearer ones by an independent method. In the 1990s this was achieved through parallax measurements made by the European Space Agency (ESA) satellite Hipparcos. The scale will be further refined when the ESA mission GAIA is launched – this would make parallax measurements far more accurate even than those made by Hipparcos.

Video credits

This video sequence has been re-edited by The Open University in 2003 from two existing BBC TV programmes produced for The Open University.

Speakers (in order of appearance)

Mike Merrifield (University of Nottingham; formerly the University of Southampton)

Adriaan Blaauw (New Leiden Observatory, and the Kapteyn Institute)

Hendrik van de Hulst (New Leiden Observatory)

Gerry Gilmore (Institute of Astronomy, University of Cambridge)

Jim Cohen (NRAL, University of Manchester)

Butler Burton (New Leiden Observatory)

Michael Hauser (NASA Goddard Space Flight Center)

Reinhard Genzel (Max-Planck-Institut für Extraterrestrische Physik)

Academic consultant – Sean Ryan (The Open University)

S282 Astronomy

Multiwavelength views of the Andromeda Galaxy

Study time: 45 minutes

Summary

In this image-based activity you will study the Andromeda Galaxy (M31) and its small companion M32 through images obtained at X-ray, visible and infrared wavelengths. You will examine the images and interpret what you see with the aim of determining the main sources of emission at these wavelengths.

You should have read to the end of Chapter 2 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- A recognition of the dominant sources of emission from a normal galaxy in these three wavebands.
- An appreciation of how different wavebands can be used to provide information on different constituents of a galaxy.

The activity

The activity uses images of the Andromeda Galaxy, M31, obtained in three different parts of the spectrum. M31 is the nearest large spiral galaxy to the Milky Way and is the most distant object visible to the naked eye (in reasonably good conditions).

The optical image, which is a negative image made using a photographic emulsion sensitive to blue light with wavelengths around 500 nm, is from the Palomar Observatory Sky Survey, carried out on the 48 inch Schmidt telescope at Palomar Observatory in California in the early 1950s. The infrared image was obtained by the Infrared Astronomy Satellite (IRAS) during its relatively short mission in 1983. This image is made in a band of wavelengths around 25 μm. The X-ray image was obtained by the German mission ROSAT (Röntgensatellit) in the early 1990s. These observations were made using a detector called the Position Sensitive Proportional Counter (PSPC) at the focal plane of an X-ray telescope.

Question 1

Why are the infrared and X-ray images obtained using satellites?

You will study each image in turn, identifying the main features and interpreting the sources of the emission at each wavelength. As well as the images, we have provided brightness contours for the infrared and X-ray images, which can be overlaid on the images to make comparisons easier.

- Start the S282 Multimedia guide, open the folder called 'Galaxies', then click on the icon for this activity (Multiwavelength views of the Andromeda Galaxy).
- Press Start to open the required set of images.

(Note that this image set is *not* available in the Image Archive.)

A window will open which displays optical image of the Andromeda Galaxy. On the right-hand side of the screen you will see a table of options that allow you to select different wavelength images and contour maps. (Note that in this image set – the term 'visible' has been used to denote optical observations at visual wavelengths. We shall refer these as the 'optical' images in these notes.)

- First, examine the optical image of M31, without any contours.

Question 2

Describe the appearance of the optical image and, using what you have learned about galaxies in Chapter 2 of *An Introduction to Galaxies and Cosmology*, identify and interpret the typical characteristics of a spiral galaxy.

The distance to M31 is about 690 kpc, so by measuring the angular extent of features in the optical image it is possible to determine their actual extent. In the next question you are asked to calculate the scale of the image and to then measure features using the on-screen ruler.

The on-screen ruler should be accessed in the following way:

- On the right-hand side of the screen there is a box that is labelled Launch Keyword Search and Measure Tool. Click on this.
- A list of all nine multiwavelength images will be displayed – for the image you wish to measure, click on the box labelled Measure image.
- The image to be measured will be displayed in a new window.
- To measure an angular extent in the image – position the cursor at one end of the feature you wish to measure. Press and hold down the left mouse button, then drag the cursor to the other end of the feature (you should see a white line extend between the two chosen points). The angular distance between the two points will be displayed.

Question 3

(a) Given that the distance to M31 is about 690 kpc, what is the scale of the image, i.e. how many pc in M31 correspond to 1 arcmin on the image? (*Hint*: see Equation 3.8 of *An Introduction to the Sun and Stars*.)

(b) Using the on-screen ruler, measure the diameter of the disc and the nuclear bulge in arcmin. Remember that the true shape of the disc is circular, we are seeing it at an angle, so you need to measure the major axes, which are not foreshortened. Estimate the size of the disc and bulge in kpc. How does this compare with the size of the Milky Way?

The apparent shape of the disc is an ellipse. If the disc is actually circular, then the angle of inclination θ to our line of sight can be found using the relation

$$\cos\theta = \text{(minor axis)/(major axis)}$$

The angle θ can be found using the $\cos^{-1}$ function which is defined in the following way.

If the cosine of an angle α has the value x, (i.e. $\cos\alpha = x$) then we can say that

$$\alpha = \cos^{-1}(x)$$

Note that '$\cos^{-1}(x)$' means 'the angle whose cosine has the value x'. It is important to realize that it *does not* mean $1/\cos(x)$.

You should be able to find the $\cos^{-1}$ function on your calculator – but note that it is sometimes called 'arccos'. (Remember that your calculator must be set up to work in degrees if you want the $\cos^{-1}$ function to give you an angle that is expressed in degrees.)

■ Use the $\cos^{-1}$ function on your calculator to find the angles (in degrees) that have the following cosines: 0.50, 0.866, 0.0, 1.0.

❑ $\cos^{-1}(0.50) = 60°$
$\cos^{-1}(0.866) = 30°$
$\cos^{-1}(0.0) = 90°$
$\cos^{-1}(1.0) = 0°$

So the angle of inclination θ can be found from

$$\theta = \cos^{-1}[\text{(minor axis)/(major axis)}] \qquad (1)$$

(A value of $\theta = 0°$ corresponds to viewing the disc face-on, whereas $\theta = 90°$ corresponds to viewing the disc edge-on.)

Question 4

Measure the major and minor axes of M31 and hence calculate the angle of inclination θ.

Now we are going to compare the appearance of M31. Examine, in turn, the infrared and X-ray images by clicking on the appropriate link in the navigation menu. (Note that the faint striped pattern running from the bottom left to top right in the infrared image is due to instrumentation effects.)

Question 5

Describe what you see in (a) the infrared image, and (b) the X-ray image including an estimate of the size of any interesting features. By looking back at what you have learned about stars, the ISM and the Milky Way, suggest what sort of sources might be causing the emission. Compare the images in all three wavelength bands and comment briefly on the similarities and differences. How does the amount of detail compare across the wavelengths? Which of the features you identified in Question 1 appear to show up in the infrared or X-ray?

As noted in the answer to Question 5, the resolution of the infrared image is much lower than that of the optical image – in fact about 60 times lower. The optical image was obtained with a resolution of about 2 arcsec – this means you should be able to distinguish two objects which are only 2 arcsec apart, whereas the resolution for the infrared image is about 2 arcmin.

Question 6

Using the results of Question 3, what are the distances between objects (i.e. within M31) that could be resolved at infrared and optical wavelengths? How do these distances compare to interstellar distances?

For observations using telescopes in general, i.e. including the infrared observations shown here, the angular resolution depends on the diameter of the main mirror (the larger the better) and the wavelength of the observed radiation (the shorter the better). Here, not only are the visual wavelengths shorter than the infrared ones, but the ground-based Palomar telescope is bigger than the space-based IRAS telescope. As a consequence of this, and other factors related to the design of the infrared detectors, the optical observations have a higher resolution than the infrared observations.

The X-ray images have a resolution of about 30 arcsec, and so are intermediate in resolution between the optical and infrared images. In the X-ray image, the close-packed sources in the galactic centre cannot be distinguished, so it's not easily possible to tell whether the emission there is diffuse or from point sources like the others.

Let's make the comparisons of the images a bit more rigorous. Start by overlaying each of the infrared and X-ray images with its own contours, so that you are clear in your mind about which contour 'islands' are peaks of brightness and which are actually 'dips', showing darker areas. For example the infrared image has elongated 'islands' around the bright ring, especially noticeable in the top left hand section, and a dark area in the disc within the ring but outside the nuclear bulge. Once you are comfortable with what the contours represent, go back and overlay each set of contours in turn on the optical image.

Question 7

With the optical image overlaid by the infrared contour, determine what the bright ring at 10 kpc in the infrared corresponds to in the optical. What do you think these features are, and why do they emit radiation in the infrared? Do you think the bright infrared emission in the nuclear bulge is due to the same sort of source?

Question 8

Now do the same sort of exercise with the X-ray contour overlay. What, if anything, do the X-ray sources correspond with at optical wavelengths? And what about comparing the infrared and X-ray images with each other? Do you find any common points?

Question 9

Finally, consider the dwarf elliptical companion galaxy, M32 (south of the nuclear bulge at RA 00h 42m 50s, Dec 40° 50′). Unlike the other companion galaxy, M110, this was well within the field of view for the IRAS and ROSAT surveys. At which wavelength(s) can you see emission from M32? What do you think it could be caused by? Why do you think it does not show up in all wavelengths?

Observing M31 for yourself

M31 is just about visible to the naked-eye – although it appears only as a faint ‘smudge’ and does require a dark observation site. You can locate M31 using your planisphere. At the beginning of July it is in the NNE at sunset, moving upwards and southwards until it is almost overhead at sunrise. Around midnight it is in the NE, with an altitude of about 20° (roughly the width of an outstretched hand at arm’s length).

Answers and comments

Question 1

The atmosphere absorbs strongly at infrared and X-ray wavelengths. From Figure 1.38 of *An Introduction to the Sun and Stars*, it can be seen that X-rays are completely blocked by the atmosphere. Although there are some ‘windows’ in the atmosphere at infrared wavelengths, at a wavelength of 25 μm the atmosphere is opaque. This means that X-ray and infrared observations (at wavelengths greater than about 10 μm) must be made from observatories in space.

Question 2

(Remember that this image is in the negative, so bright features look dark and vice versa.) This image shows a largish bright nuclear bulge set in a less bright disc. The outline of the disc appears to be elliptical, which implies that we are viewing it at quite a large angle of inclination. There is a clear bright spiral arm running up the left-hand side of the disc and around the top, and darkened lanes running down the right-hand side and around the bottom, where they are interlaced with bright arms. There are two bright dwarf elliptical companion galaxies, one appearing just below the nuclear bulge and the other towards the top right of the image. The rest of the small bright points around the image are foreground stars in the Milky Way.

The brightness of the bulge is due to the high density of stars that have a peak of their emission in the optical part of the spectrum. We can't really see any more structure in the bulge at this wavelength. The bright spiral arms are due not to the enhanced density, but to the luminous objects in the arms associated with star formation, such as HII regions, open clusters and OB associations (*An Introduction to Galaxies and Cosmology* Section 1.4.4). The rest of the disc is more dimly illuminated by stars. The darkened lanes are likely to be due to dust which obscures the visible light (*An Introduction to Galaxies and Cosmology* Section 1.2.3, compare also Figure 1.1).

Question 3

(a) The radius R of a distant object, in terms of half the angle α it subtends and its distance d is given by *An Introduction to the Sun and Stars* Equation 3.8

$$R = [(\alpha/2)/\text{radians}] \times d$$

Here, we want the distance D within M31 corresponding to 1 arcmin, which is therefore given by

$$D = [(1\ \text{arcmin})/\text{radians}] \times d$$

where $d = 690$ kpc.

Since 60 arcmin = 1 degree and 57.3 degrees = 1 radian,

1 arcmin = $1/(60 \times 57.3)$ radians = 1/3438 radians, so

$$D = (690/3438)\ \text{kpc} = 0.20\ \text{kpc}$$

So, 1.0 arcmin on the image is equivalent to 0.20 kpc.

(b) I obtained a value of around 120 arcmin for the diameter of the visible disc in the image, so the actual diameter is about 120 arcmin $\times$ 0.20 kpc arcmin^{-1} = 24 kpc. This gives a radius of 12 kpc, which is a little smaller than the visible disc of the Milky Way at 15 kpc. However, since the edge of the disc of M31 comes quite near the edge of the image, it is possible that the disc could have a greater extent than the image.

For the bulge, I obtained a diameter of 42 arcmin, which is equivalent to 42 arcmin $\times$ 0.20 kpc arcmin^{-1} = 8.4 kpc, this time a little bigger than that of the Milky Way at 6 kpc. (Remember that M31 is classified as Sb, whereas the Milky Way is SBbc, or similar, so we would expect M31 to have a relatively slightly bigger bulge.)

Question 4

In Question 3, the major axis of the disc of M31 was found to have an extent of about 120 arcmin. The minor axis of the disc image is about 30 arcmin across. The angle of inclination θ is given by Equation 1

$$\theta = \cos^{-1}[(\text{minor axis})/(\text{major axis})]$$

$$\theta = \cos^{-1}[(30\ \text{arcmin})/(120\ \text{arcmin})] = \cos^{-1}[0.25]$$

$$\theta = 76°$$

(*Comment*: given that the measurements of the major and minor axes are approximate, an appropriate way of quoting this result would be that the angle of inclination is between 70° and 80°.)

Question 5

(a) The most obvious features of the infrared image are a very bright region in the centre of the image, surrounded by much fainter emission, with a slightly branched bright elliptical ring at about 50 arcmin from the centre. The emission looks as though it extends beyond the top right and bottom left corners of the image, so the whole disc may well be bigger than the estimate we made in Question 3. There is also a small emission source towards the bottom left of the image. This doesn't correspond in position to either of the dwarf galaxies.

The image is much less detailed than the optical image, it is clearly at a much lower resolution – it is not possible to distinguish small features.

Infrared emission in astronomical sources typically arises from dust, which absorbs some of the visible and UV light from stars and HII regions and re-emits it at longer wavelengths. This dust occurs in the ISM and in circumstellar shells around old red giant stars.

(*Comment*: in order to emit so strongly at a wavelength of 25 μm, the dust must be heated to relatively high temperatures – above 200 K. This is hotter than most interstellar dust, which consists of relatively large grains at temperatures of below 100 K. However, very small dust grains or very large molecules can be heated to temperatures above 200 K after absorbing a single ultraviolet photon, and it is this material that gives rise to the emission seen at 25 μm.)

(b) The X-ray image is quite different to the optical and infrared images, in that there is very little sign of the disc. Like the infrared image, there is a bright source at the centre, but this time what we see around are a large number of smaller bright sources, with no real sign of any sort of diffuse emission over the whole galaxy. There may be some diffuse emission in the bright centre, but it is difficult to distinguish whether or not it is just due to the point sources running into each other.

By comparison with the X-ray map of Milky Way (*An Introduction to the Sun and Stars* Figure 9.16) the bright sources are likely to be interacting binaries containing neutron stars.

(*Comment*: of the 400 or so sources in the region of M31 identified during the ROSAT survey, 30–40% are thought to be either foreground or background sources, several were identified with known globular clusters in M31 and a few with supernova remnants. However the majority of the small sources in the image are thought to be interacting binaries, though in the bulge there may be some diffuse thermal emission from very hot gas. The emission from globular clusters arises from the interacting binaries that they contain.)

Question 6

2 arcmin in the image corresponds to about 0.4 kpc, or 400 pc. This is a much larger distance than that of nearly all the 100 brightest stars in Appendix A4 of *An Introduction to the Sun and Stars*. An angular resolution of 2 arcsec corresponds to (400/60) pc = 7 pc which is still further than any of the 100 nearest stars in Appendix A3 *of An Introduction to the Sun and Stars*, but a good enough resolution to distinguish far more detail – such as individual HII regions.

Question 7

With the infrared contours overlaid on the optical image, you can see that the infrared bright ring at 50 arcmin corresponds both to the star forming regions of the bright spiral arms and to the dark dust lanes you observed in the optical image. The bright regions correspond to HII regions where the new young stars are heating their surroundings. The dust lanes are likely to be obscuring similar HII regions. Much of the disc doesn't emit so strongly in the infrared and doesn't show clear dust lanes at visible wavelengths, so presumably contains either less dust, (or, as noted above in the answer to Question 5, dust with larger grains), or doesn't have such a strong heating mechanism. It's also clear that the very bright area in the centre does correspond to the centre of the bulge with its dense population of stars, so it seems likely that much of the bright emission from the centre of M31 is due to emission from circumstellar shells which exist around giant stars.

(*Comment*: the isolated source towards the bottom left corresponds to a foreground (Milky Way) star which is a red giant surrounded by circumstellar dust.)

Question 8

Overlaying the X-ray contours on the optical image confirms that not many of the X-ray sources correspond to anything at optical wavelengths. We would not expect to be able to see enough detail in this optical image to be able to distinguish binary stars, so would not expect to be able to spot a direct correspondence. Again the bright centre clearly corresponds to the centre of the bulge, as discussed above this may be partly due to very hot gas within the bulge.

The other obvious correlation is with the dwarf galaxy M32, see Question 7.

There is very little in common between the infrared and X-ray images apart from the bright galactic centre. Since the emissions arise from quite different sources this is to be expected.

Question 9

M32 shows up clearly at both optical and X-ray wavelengths, but not in the infrared. Being an elliptical galaxy it contains many stars, shining at optical wavelengths, but very little gas or dust, so there is no mechanism for emitting in the infrared. The X-ray emission arises, at least in part, from a number of interacting binary stars.

(*Comment*: it has been also suggested that M32 may contain a massive central black hole, which might give rise to some the X-ray emission from an accretion disc around it, but the evidence to support such a claim is not strong.)

S282 Astronomy

The AGN ‘zoo’

Study time: 60 minutes

Summary

In this activity you will make use of an online database of galaxies called the NASA/IPAC Extragalactic Database (NED). This database contains a wealth of material – including photometric measurements, spectra, images and maps – of several million galaxies. You will extract spectra and images of some active galaxies which are representative of the types you have met in the course.

This activity will require you to connect to the Internet for the whole session – about 1 hour.

You should have completed Section 3.3 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Use a web browser to gather information from an online database.
- Recognize the different ways of plotting the broadband spectrum of an object and their relative advantages.
- Appreciate how broadband spectra can be used to help distinguish different types of galaxy.

Background to the activity

Astronomers were among the first scientists to see the potential of the World Wide Web not only for exchanging information but for providing access to the many databases of observational results. The NASA/IPAC Extragalactic Database, commonly known as NED, is run jointly by the US National Aeronautics and Space Administration (NASA) and the Infrared Processing and Analysis Center at the California Institute of Technology (Caltech).

Through NED astronomers can access a large amount of observational data about galaxies and access the original research papers in which the data were published.

This site is characteristic of scientific websites designed for professional use in that it is not particularly user-friendly. Consequently you may find that some aspects of this website are difficult to understand. For that reason we will only look at a few of the facilities offered by NED. When you are more confident you could come back and explore further by yourself.

Before starting the activity, make sure you are familiar with how the broadband spectra of galaxies are plotted (*An Introduction to Galaxies and Cosmology* Section 3.2.2) and the different kinds of flux units (*An Introduction to Galaxies and Cosmology* Box 3.2).

The activity

- Start up your web browser and connect to the Internet.
- Go to the NED homepage at http://nedwww.ipac.caltech.edu
- The panel at the bottom of the page gives information about the number of objects in the database.
- How many objects are in the database?
- ❑ When this activity was written there were 163 million objects. The numbers may well have changed since then.

To start with, we are going to look at the spectral energy distributions (SEDs) of some of the types of object you have come across so far in Chapter 3.

The main part of the NED homepage is the table with five columns (labelled ‘OBJECTS’, ‘DATA’, ‘LITERATURE’, ‘TOOLS’, ‘INFO’). Links in the cells of this table take you to different kinds of information.

- In the column headed DATA click on the link Photometry & SEDs. This will take you to a new page headed ‘Photometric Data Search Based On An Object Name’.

The first galaxy we will look at is a spiral galaxy called Messier 83 (M83), a type Sc spiral.

- In the box labelled ‘Enter object name’ type `M83`.
- Leave the next three boxes as they are. (In order, they should read: ‘Data as Published and Homogenized (mJy)’, ‘Error Bars’, and ‘No Point Labels’.)
- The fifth box allows you to choose either frequency or wavelength units for the horizontal axis (x-axis) of the SED. In S282 we have been using wavelength units, so click on the arrow to the right of the fifth box and select X=log(Wave.)(microns) from the drop-down list. When the SED is plotted it will have a scale on the horizontal axis which is the logarithm of the wavelength in micrometres.
- The sixth box allows us to specify the flux units. For now, select Y=log(Fnu)(W/m2/Hz) from the drop-down list. This will produce a scale on the vertical axis which is the logarithm of the spectral flux density (F_ν) in units of watts per square metre per hertz (W m^{-2} Hz^{-1}).
- The final box should be left as it is (‘Fixed data range (for comparisons)’).
- Click the Photometry button and wait.

You will now get a long page of data with a graph at the top. Information about each data point and its source is given in the table below the graph. For the purposes of this activity just concentrate on the graph at the top of the page.

- What is the title of the graph?
- ❑ It should say ‘Spectral Energy Distribution (SED) for MESSIER 083: 149 Data Points’. This shows that NED has retrieved the data for the correct object (note that the number of data points may be different if the database has been updated since this activity was written).

■ What are labels on the horizontal axis (at the bottom) and the vertical axis (to the left)?

❑ The horizontal axis is the logarithm of the wavelength (λ) in μm and the vertical axis is the logarithm of the spectral flux density (F_ν) in W m^{-2} Hz^{-1}. So the units are as we requested.

■ Express the wavelength range of the spectrum in metres (to the nearest factor of 10). What part of the spectrum does the graph cover?

❑ On the left, the log of the wavelength is –4, so the wavelength is 10^{-4} μm = 10^{-10} m. On the right, the log is 8, so the wavelength is 10^{8} μm = 10^{2} m. These wavelengths range from X-rays through to radio waves.

The scale on the top of the graph gives us a bit of extra useful information. On the left, '1 keV' tells us that the photon energy is 1 keV, which confirms that these are X-rays. U, V, J, K and L are standard photometric bands (U is ultraviolet, V is visual and J, K and L are near-infrared), while '*IRAS*' refers to the wavelength range of the Infrared Astronomical Satellite which surveyed the sky in 1983. '1 mm' is, of course, in the millimetre-waveband and '6 cm' is at radio wavelengths. 'HI' marks the wavelength of the 21-cm spectral line of neutral hydrogen. Finally, '100 MHz' is a radio frequency in the middle of the VHF (FM) broadcast band.

■ Although hard to see, if you look carefully you should be able to see that some of the plotted points have horizontal or vertical bars through them, and some have both. What do they mean?

❑ These are error bars. They tell us the uncertainty in the wavelength or spectral flux density for each measurement.

Now that you understand what the graph is showing, we can start to look at the data itself.

■ This galaxy emits in X-rays and also in radio waves. In which of those two regions does it emit more strongly?

❑ According to this graph the galaxy emits about 10^{6} times more strongly in radio waves than in X-rays.

If you have read Chapter 3 of *An Introduction to Galaxies and Cosmology* you may be suspicious about this conclusion! And you would be right, because plotting *spectral flux density* does not give us a fair representation of where the galaxy's energy is being emitted. In Section 3.2.2 of *An Introduction to Galaxies and Cosmology* you learned that we should be plotting $\log(\lambda F_\lambda)$ instead. NED does not allow the option to plot $\log(\lambda F_\lambda)$, but it does let us plot $\log(\nu F_\nu)$ which is exactly the same thing (see Question 1 at the end of this activity).

■ To the right of the graph (you may have to use the horizontal scroll bar) are the boxes for changing the plot. Select Y=log(NuFnu)(W/m2) and click on the Plot Again button to get a new graph.

You should now see the same data plotted with $\log(\nu F_\nu)$ (in W m^{-2}) on the vertical axis.

■ How do the X-ray and radio energies compare now?

❑ The galaxy emits about 10^{3} times more energy in X-rays than in radio waves.

This is a truer representation of the importance of different regions of the spectrum.

Finally, look carefully at the part of the SED between the ultraviolet and the 1 mm mark. The first thing that you will probably notice is that at some wavelengths there appear to be many different values of flux density. For instance, in the J-band there is a very wide range flux densities. This illustrates an important point about this database – it is a source of real astronomical data taken over a long time interval. Some of the differences in flux density at a given wavelength could be due to intrinsic variability of the source, but they could also be due to differences in calibration between different observers. This is especially a problem in the near-infrared – where the calibration of some of the older data is rather uncertain – and this, rather than variability of the source, accounts for the scatter in the data in this SED.

Rather than getting too concerned about the spread in the data points at any wavelength, try to view these SEDs as if you had to draw a schematic smooth curves through the data – much in the way as is done in Figure 3.11 of *An Introduction to Galaxies and Cosmology* (the spectrum of NGC 7714).

■ Examine the SED of M83 between the ultraviolet and millimetre wavelengths. How could you describe the shape of the spectrum in this region?

❑ It seems to be double peaked. There is one peak around 1 μm and another around 100 μm.

■ What could be causing these two peaks? What kind of galaxy might this be?

❑ The peak around 1 μm is due to starlight, the peak around 100 μm is probably due to interstellar dust.

In fact, this spectrum is very similar to that of NGC 7714 (*An Introduction to Galaxies and Cosmology* Figure 3.11). M83 is an example of a *starburst* galaxy.

You may want to save the SED that you generated, so that you can refer to it again, or compare it with those of other galaxies. The simplest way to do this is as follows:

- Right click on the blue area of the SED plot. (Note that if you left click on the plot – a new window will open which zooms in on the part of the spectrum that you have selected.)
- From the menu that appears select the option Save Picture As....
- In the 'Save Picture' dialog box, enter a name for the plot (e.g. `M83_SED`). The 'Save as type' list box gives two options – you can save the plot either in GIF or BMP format. (It is likely that BMP format would be the most useful since such files can be viewed and edited using the 'Paint' application in Microsoft Windows.)

Of course, you can repeat this process for any SED that you generate using NED.

Next we are going to look at an elliptical galaxy.

■ Given what you know about the composition of elliptical galaxies, what would you expect their SEDs to look like?

❑ Ellipticals have very little gas and dust. We would expect their SED to look like a composite of stellar spectra only.

One of the best known ellipticals is M32, a companion to the Andromeda Galaxy.

- Use your browser's Back button to return to the page called 'Photometric Data Search Based On An Object Name'. This time, enter `M32` as the object name and select a log(νF_ν) plot with a wavelength scale as before (i.e. select X=log(Wave.)(microns) and Y=log(NuFnu)(W/m2)).
- Click on the Click the Photometry button and you should now see an SED of M32.

This SED has a broad peak in the optical–infrared region.

■ What do you think the downward pointing arrows in the IRAS and radio parts of the spectrum mean?

❑ They are upper limits. It means that the galaxy has not been detected at that flux density.

This indicates that normal elliptical galaxies such as M32 have stellar spectra with insignificant radiation at very high or very low energies. Unlike M83, there is no detectable emission from dust in this galaxy.

Now we are going to look at a quasar, a type of active galaxy.

- By following similar steps as you used to generate the SED of M32, obtain the SED of the quasar 3C273. To do so, you should enter `3C273` in the object name box on the page called 'Photometric Data Search Based On An Object Name'. Make sure you specify a wavelength scale and a log(νF_ν) plot.

This famous quasar was the first to be discovered. Compare the plot you are now looking at with Figure 3.10 in *An Introduction to Galaxies and Cosmology* – you should be able to see that they are very similar. Looking at the SED produced by NED you may also have noticed a warning message printed on the plot that says 'Data outside fixed range. Plot again using autoscaling.' This means that there are some measurements that fall outside the limits of the wavelength range that has been used. To see all the data, you should re-plot the SED as follows:

The lowest of the five boxes on the right-hand side of the SED provides a choice between 'Fixed data range (for comparisons)' and 'Autoscale data range'. Select the latter option, and press the Plot Again button.

■ In what part of the electromagnetic spectrum is the additional data that is shown on this SED?

❑ The additional data point is at log(λ/μm) ~ –9, or $\lambda \sim 10^{-15}$ m. This is an extremely energetic γ-ray.

This SED shows that a substantial part of the total luminosity of 3C 273 is emitted at extremely high photon energies.

Here are some other examples of active galaxies that you might like to inspect and compare. (In each case it may help to save a plot of the SED as described earlier so that you can view several SEDs at once.)

NGC 1068 – a nearby (type 2) Seyfert galaxy.

BL Lac – the original blazar to be discovered

M84 – the radio galaxy pictured in Figure 3.21 of *An Introduction to Galaxies and Cosmology*, which lies in the Virgo cluster.

■ Both M84 and M32 are elliptical galaxies. What is the main difference between their SEDs?

❑ The SEDs are very similar in the visible and near-infrared, but M84 is bright in X-rays and also has considerable radio emission.

The X-ray and radio emission come from the active galactic nucleus. Otherwise their SEDs are similar.

Finally, while you have the M84 plot on the screen, we want to show you how to access more information about specific objects.

- Click on the link to MESSIER 084 in the title of the SED. You will now see a page of information about M84.

The first section contains the position and redshift information. The second section lists the many other designations for M84 from various catalogues and surveys. At the bottom of the page are links to other databases where more information can be found.

- Scroll back to the point on the page where there is a small image of the galaxy (this is a negative image with the sky shown as white).
- Click on the images link (to the left of the image). It may take a few minutes for the page to finish downloading as the many images amount to a little over a megabyte.

The large table you now see gives information about each of the images in the NED database. It shows a preview image on the left and, among other things, lists the wavelength at which the image was made, the telescope used and a reference to the paper where it was first published. Many of these images use a format called FITS (which stands for 'Flexible Image Transport System') which is in widespread use in the astronomical community but is not supported by web browsers. For example, the image labelled '285KB FITS image' is very similar to the radio map shown in *An Introduction to Galaxies and Cosmology* Figure 3.21. The table shows it was made with the VLA – the Very Large Array radio telescope in New Mexico.

- Your browser will, however, display images marked 'JPG'. For example, click on the image labelled '45KB JPG image' that is from the 'Einstein_Obs'.

This is a map of X-ray emission from M84, made with the Einstein X-ray Observatory. The X-ray contours are superimposed over a photograph of the galaxy. (NGC 4374 is one of the alternative names for M84, as you can check by looking back at the page of information you downloaded earlier.) You can see that the emission is concentrated on the nucleus of the galaxy which is the source of the energy.

Thus NED provides professional astronomers with a very quick and relatively easy way to review what is known about a particular galaxy, and is an invaluable tool for planning further observations.

There is much more to explore in NED, but we have now finished with the online part of this activity.

Question 1

The spectral flux densities F_λ and F_ν are related by the expression

$$F_\lambda = \frac{c}{\lambda^2} F_\nu \qquad (1)$$

(a) Show that the quantity λF_λ is equal to νF_ν.

(b) Show that the units of λF_λ and νF_ν are those of flux density.

Question 2

All the SEDs you have seen here and in Chapter 3 have gaps between the X-ray and near-UV parts of the spectrum, just where the 'big blue bump' is said to be. Can you think why that is?

Answers to questions

Question 1

(a) We multiply both sides of Equation 1 by λ and note that $\nu = c/\lambda$.

$$\lambda F_\lambda = \lambda \frac{c}{\lambda^2} F_\nu = \frac{c}{\lambda} F_\nu = \nu F_\nu$$

(b) Putting the units into SI we find:

For λF_λ m × W m^{-2} m^{-1} = W m^{-2}

For νF_ν Hz × W m^{-2} Hz^{-1} = W m^{-2}

So both quantities have units of watts per square metre, which are the units of flux density.

Question 2

The gaps suggest that measurements have not been obtained in that region, which is known as the extreme ultraviolet (EUV or XUV). You might guess that this part of the spectrum is not observable from the ground, since EUV waves are absorbed by the Earth's atmosphere. That is true but it is not the whole story. Radiation of 10–100 nm is also absorbed by the neutral hydrogen of the interstellar medium and that means our EUV view of the sky is very restricted. Although there are some gaps in the interstellar medium that do allow more distant objects to be seen, it is practically impossible to make useful extragalactic observations in the EUV part of the electromagnetic spectrum.

S282 Astronomy

Jets and black holes

Study time: 40 minutes

Summary

This activity relates to a video sequence in which you will consider astrophysical jets. Jets were discovered in connection with active galaxies, but are now recognized to occur on a wide range of scales. The sequence concentrates on the observational techniques that are used to study this energetic phenomenon, and considers the link between jets and black holes. You should have completed Chapter 3 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Appreciate the way that radio and millimetre-wave telescopes are used to investigate jets and associated structures in a wide variety of astronomical objects.
- Recognize that jets occur in a variety of astrophysical objects.

The activity

This programme brings together a discussion of the outflow from star-formation regions (mentioned in Chapter 5 of *An Introduction to the Sun and Stars*), jets from evolved stars, and the jets associated with active galaxies (the subject of Chapter 3 of *An Introduction to Galaxies and Cosmology*). The observing techniques featured are those of radio astronomy, particularly radio interferometry.

The activity is based around a long (about 24 minute) video sequence which was originally broadcast as a television programme in 1997. Although there have been some advances in observing technology since the video was made, the principles behind the observations are still relevant today. Furthermore, the mechanism of astrophysical jet formation remains a mystery.

- Start the S282 Multimedia guide and then click on Jets and black holes under the ‘Galaxies’ folder in the left-hand panel.
- Press the Start button to run the video sequence.

After you have watched the video sequence, read the summary provided in the ‘Notes’ below.

Notes

In some stars and galaxies we observe an interesting and important phenomenon – jets of material streaming out of these bodies. In many cases these jets can only be observed at long wavelengths, and require observations made with higher spatial resolution than is normally available with a ground-based optical telescope. Until we understand them (and we are far from doing so) our knowledge of stellar and galactic evolution will remain partial and precarious. This video sequence addressed these points, in four sections.

1 Young stars are observed in dense clouds, which are assumed to be contracting under self-gravitation, though this process is never directly observed. A common feature of such star-forming regions is *expulsion* of matter in symmetrical (back to back) jets which are known as bipolar outflows (*An Introduction to the Sun and Stars* Section 5.3.4). They are typically about a parsec long (similar to the distance from us to Alpha Centauri) and carry away matter at about $10^{-6}M_0$ a year, so they must obviously be short-lived. We saw how the James Clerk Maxwell Telescope (JCMT), operating at millimetre and submillimetre wavelengths is able to make detailed images of such objects. By also recording spectra one can find how the Doppler shift of a strong spectral line (a rotational transition of the carbon monoxide molecule in the case shown) varies across the object: the speed and collimation of the outflow is clearly revealed.

   Many such bipolar outflows have been seen (see, for example, *An Introduction to the Sun and Stars* Figure 5.13) and they are compatible with the idealized visualization shown in *An Introduction to the Sun and Stars* Figure 5.12. Notice the presence of a disc or torus at the centre (which would be called an accretion disc in other contexts).

   There is only a hint of it in the JCMT image shown in the programme, but such discs are thought to be associated with all types of jet. The nature of the driving force is unknown.

2 The next step in the story is the star SS 433 at the centre of a nearby supernova shell W50 – a star now identified as a spinning and wobbling compact star, which accretes material from a binary partner, presumably into an equatorial disc (see the illustration of a similar type of binary system in *An Introduction to the Sun and Stars* Figure 9.15), and somehow directs it into fierce *axial* jets. This description is almost identical with that of bipolar outflow, but the length of the jets in SS 433 is much larger – extending some 50 pc from the central object into the surrounding interstellar medium.

These jets, and their precession, were first identified from optical spectra that had the extraordinary feature of containing both red-shifted and blue-shifted emission lines. The shifts varied with a period equal to the precessional period. Radio imaging at very high (milli-arcsec) resolution revealed the very innermost section of the jets, where the radio emission comes from blobs rather than the whole jet. This enabled a direct measurement of transverse jet speed, to compare with the Doppler shifts due to radial speed (with respect to us). Both give speeds in the region of a quarter that of light! The jets as revealed by the visible light images, and the radio lobes, are vastly bigger than the innermost section shown in the high-resolution radio image sequence. As in the case of bipolar outflows, both the source of the power, (much greater in this case), and the very effective collimation mechanism, are still mysteries. The SS 433 jets are enormously long in comparison with the tiny compact star, which is thought to be a neutron star and therefore only a few kilometres in diameter.

SS 433 is one of several binary stellar systems in our Galaxy that have jets. Because these relatively close objects can be studied in detail they may reveal processes common in many other types of accretion phenomenon.

3 Whatever the mechanism operating in SS 433, something similar seems to apply at the much bigger scale of the cores of active galaxies. In studying distant galactic centres astronomers benefit greatly from the very high resolution afforded by arrays of radio dishes. One such array, MERLIN (the Multi Element Radio Linked Interferometric Network), has seven dishes spread over England, directly linked to the Jodrell Bank Observatory. The video sequence showed the characteristic period variation of the *interference* of signals from a pair of dishes as the Earth rotates the baseline relative to the source. We saw a simulation (made at the VLA (Very Large Array) by Rick Perley) of the way the signals of many pairs can be correlated to reveal fine structure in the source object. You saw a typical double radio source (a twin-jet), Cygnus A, as an example (see *An Introduction to Galaxies and Cosmology* Figure 3.20). A refinement of this technique, using very precise local timing and subsequent correlation rather than direct real-time links, allows the use of very long (intercontinental) baselines. (That's how high resolution images of the moving blobs in SS 433 were obtained.) This is called *very long baseline interferometry* (VLBI).

It has been known for a long time that many galaxies have violent activity in a compact core – an active galactic nucleus. This shows itself in a variety of ways, but many astronomers think that these represent much the same sort of structure seen from different angles, rather than a multiplicity of different structures and mechanisms. The hypothetical 'unified model' consists of a torus of matter leaking into a tiny accretion disc, which is heated to X-ray energies as it spirals into a very massive black hole, of mass equivalent to perhaps a hundred million stars (see *An Introduction to Galaxies and Cosmology* Figure 3.32).

In this model, an axial *pair* of jets emerges from a region close to the black hole. The mechanism by which material is accelerated into jets is not well understood. As described in Chapter 3 of *An Introduction to Galaxies and Cosmology*, the physical mechanisms that may play a role in jet formation are the magnetic fields in the accretion disc and radiation pressure effects close to the black hole.

In fact *only one* jet is normally observed. It is thought that the advancing jet – the jet that is heading in roughly our direction – is visible because it is greatly brightened due the process of relativistic beaming. Mostly, the jets are not particularly long in galactic terms – say a few times the thickness of the galaxy – hence the need for high-resolution imaging. There are several hypotheses for the guiding mechanism for the jet, and for the method by which electron energy is replenished along the jet, so that the electrons sustain the synchrotron radio emission.

4 In those jets which are *very* long there must be some mechanism that constricts and guides the outflowing material. Such jets may be hundreds of kiloparsecs or even megaparsecs long. They end in clouds of radio emission often called lobes, as we saw in the case of Cygnus A. Sometimes the jets are strongly curved, perhaps by interaction with an intergalactic medium, or by movement of the source during the millions of years of its existence. Martin Rees summarized the great advance that is waiting to be made when the processes occurring in jets can be explained.

Video credits

Presenter – Barrie W. Jones (Open University)

Speakers (in order of appearance)

Ian Robson (JCMT Hawaii, and University of Central Lancashire)

Peter Wilkinson (NRAL, University of Manchester)

Rick Perley (Very Large Array, New Mexico)

Meg Urry (Space Telescope Science Institute, Baltimore)

Andrew Wilson (Space Telescope Science Institute, Baltimore)

Duccio Macchetto (Space Telescope Science Institute, Baltimore)

Martin Rees (University of Cambridge)

Producers – Tony Jolly (BBC) and Cameron Balbirnie (BBC)

Course Team consultants – Alan Cooper, David Clark (SERC), Barrie W. Jones

S282 Astronomy

Optical and X-ray images of clusters of galaxies

Study time: 45 minutes

Summary

In this activity you will analyse optical and X-ray images of five nearby clusters of galaxies. You will compare the appearance of clusters of galaxies in these two wavebands. You will also consider how clusters vary from one another on the basis of their appearance in such images.

You should have read to the end of Section 4.4 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Appreciate the difference in appearance between different cluster types.
- Understand the spatial relationship between cluster galaxies and the intracluster medium.
- Appreciate why clusters are frequently more easily identifiable by their X-ray emission than by visual appearance.

Background to the activity

The five clusters that have been chosen for this activity are well-known nearby rich clusters, all within 200 Mpc of our own Galaxy. The clusters were first documented as part of George Abell's original study as described in Chapter 4 of *An Introduction to Galaxies and Cosmology*. The Abell cluster catalogue numbers are given, together with additional details, in Table 1.

Table 1 Cluster data.

	Abell 2199	**Centaurus**	**Coma**	**Hydra**	**Perseus**
Distance/Mpc	180	52	110	60	90
RA	16 h 29 min	12 h 49 min	13 h 00 min	10 h 37 min	03 h 20 min
Dec	+39° 33′	−41° 19′	+27° 58′	−27° 30′	+41° 30′
Abell Number	A2199	A3526	A1656	A1060	A0426

The optical images of the clusters are taken from the Digitized Sky Survey (DSS). Archived photographic plates from the Palomar Optical Sky Survey and others have been digitized using a high-resolution scanner, both preserving the data and making it far more widely accessible.

The X-ray images are from the ROSAT (Röntgensatellit) X-ray Observatory. This was a joint German, UK and US mission that was launched in 1990 and operated until 1999. The images were made using an instrument called the Position Sensitive Proportional Counter (PSPC) that had a 2 degree field of view and a spatial resolution of approximately 30 arc seconds.

By contrast, the DSS optical images presented here have a resolution approximately four or five times higher than the X-ray data. The original scans were to a much higher resolution still, but the images used here have been scaled down in order to fit the whole image on screen. Even so, the optical images are visibly more detailed than the X-ray images, and you should bear the difference in resolution in mind when comparing the images.

Part 1 A comparison of optical images

This first part of the activity concentrates on the visual aspects of the DSS images. You will compare similarities and differences between the Digitized Sky Survey images of the five clusters.

- Start the S282 Multimedia guide and open the folder called ‘Galaxies’, then click on the icon for this activity (Optical and X-ray images of clusters of galaxies).
- Press the Start button to access the set of cluster images. Images of different clusters can be selected using the buttons that are displayed on the right-hand side of the screen (Figure 1).

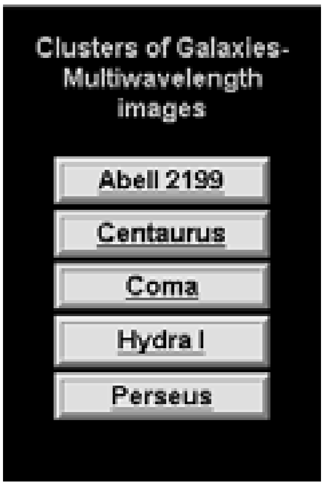

Figure 1 The selection buttons for the cluster images.

You are going to start by looking at the *optical* images for each of the five clusters. (Note that in this image set the term ‘visible’ has been used to denote optical observations at visual wavelengths. We shall refer to these as the ‘optical’ images in these notes.)

- Start by clicking on the Abell 2199 button at the top of the list on the right-hand side of the screen. An optical image of the Abell 2199 cluster will be displayed in the left-hand pane.

The first thing to notice is that the image is presented as a *negative*. This is the view that you would get if studying traditional photographic plates on a light box. Take a while to look carefully at the image and get used to the appearance of the different objects in the image.

- Now click on the other four buttons to display the Centaurus, Coma, Hydra I and Perseus clusters in turn, and note the appearance of these clusters. In each case, take a moment to study the image and note the different types of objects present in each image.

(Note that there are slight differences in the image scales: the images of the Centaurus, Coma and Hydra clusters are all 1.0 degrees to a side, whereas the Perseus image is 1.5 degrees across and the image of A2199 is 0.75 degrees wide.)

The images contain many foreground stars within our Galaxy and it is important to be able to distinguish between these and galaxies.

- Can you identify which of the objects in each image are galaxies and which are stars? How would you describe the differences between the appearance of the stars and galaxies?

- ❑ The galaxies are extended objects with bright cores surrounded by diffuse regions, whereas the majority of the stars are point-like objects. Many of the galaxy images are non-circular in outline, whereas stars should appear circular. Elliptical galaxies will always appear as either diffuse circles or ellipses. Spirals may appear face-on, edge-on or at an angle.

Brighter stars can appear as extended sources, but these can be distinguished by a sharper edge and the presence of diffraction spikes which give the star a cross-like appearance. The difference between bright stars and galaxies can be seen most easily in the image of Hydra I. Here the central region contains two very bright stars (see Figure 2), along with several spiral and elliptical galaxies of differing shapes and sizes. A large number of fainter stars appear as sharp points.

When comparing galaxies from different clusters, it is necessary to take into account the relative distances of the clusters as shown in Table 1. There are also small differences in the angular scales of the images.

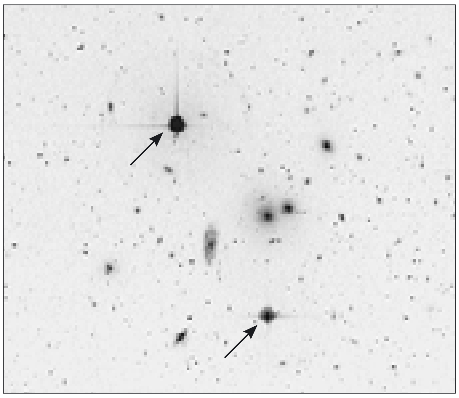

Figure 2 Foreground stars in the image of the Hydra I cluster.

Question 1

Because the clusters are at different distances, and some of the images have differing angular scales, galaxies of the same actual size will appear at different (angular) sizes on the different images. In order to compare the images properly, it is necessary to know the physical scales of the images.

Using the information in Table 2, calculate the width (in Mpc) of the field of view at the distance of each cluster and hence complete the table.

How do these distances across the field of view compare to the Abell radius. What implications does this have for your comparison of the clusters?

Table 2 Fields of view for the clusters of galaxies in the image set.

	Abell 2199	Centaurus	Coma	Hydra I	Perseus
distance/Mpc	180	52	100	60	90
angular width of image/degree	0.75	1.0	1.0	1.0	1.5
width of the field of view/Mpc					

The importance of allowing for the different scales of the images that you worked out in Question 1 is highlighted in the following example. Most of the galaxies appear quite small in the image of Abell 2199. This is largely due to the fact that, at 180 Mpc, Abell 2199 is the most distant of the five clusters. If the images of Abell 2199 and the Hydra I cluster are re-sized so that they have the same scale (Figure 3) then a comparison between the galaxies in the two clusters can be made. The central galaxy of Abell 2199 (NGC 6166) is a supergiant elliptical galaxy, and is actually much larger than the central galaxies in the Hydra image.

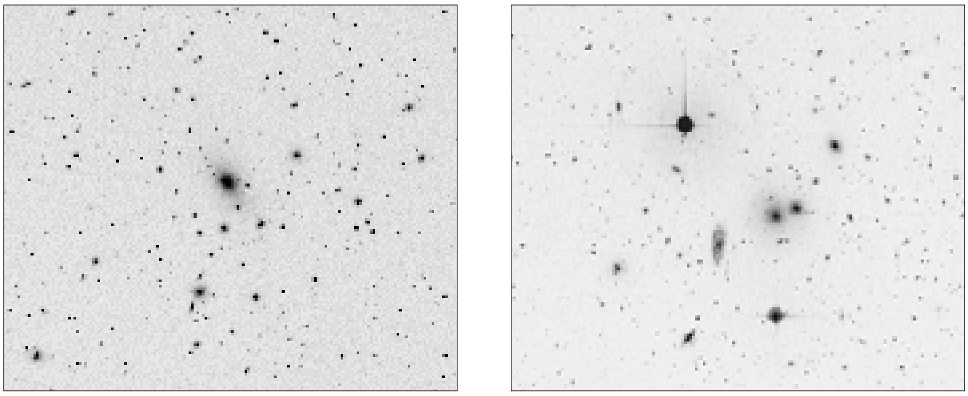

Figure 3 Comparison of central regions of the (left) Abell 2199 and (right) Hydra I clusters. The image of Abell 2199 has been corrected for distance and scale to enable the relative sizes of the galaxies to be compared. At 20 times the diameter of the Milky Way, NGC 6166 is one of the largest known galaxies.

Question 2

Using the selection buttons review each cluster image again in turn, starting with A2199. In each case, write one or two brief sentences describing the overall appearance of the cluster. Describe the relative sizes of the fields of view (Question 1), and the distribution and types of galaxies present.

As mentioned in Chapter 2 of *An Introduction to Galaxies and Cosmology*, galaxies come in a wide range of sizes: most of the clusters appear to have just one or two very large central galaxies with numerous smaller ones.

- Are all of the smaller galaxies visible in the images genuine cluster members? How would you determine whether any of the galaxies are actually *field galaxies* that only appear in the image through being on the same line of sight as the cluster?

- ❑ Field galaxies are galaxies that are not physically associated with the cluster and actually lie at different distances. Since galaxies vary greatly in size, it is difficult to tell the distance of a galaxy from its (angular) size on the image. The best way to determine the distance of a galaxy is to measure its redshift, so spectral measurements could be used to identify field galaxies and separate them from the genuine cluster members.

Part 2 A comparison between optical and X-ray images

Now we are ready to start comparing the optical and X-ray appearances of the clusters. In this part of the activity we will concentrate on the general differences between the images taken in the optical and X-ray wavebands for all five clusters.

The Navigation Menu (Figure 4) on the lower right-hand side of the screen allows you to select four different views of each cluster. The left-hand column, headed Image allows you to choose between optical or X-ray views of the cluster. In the right-hand column you can select the same two views with a contour plot of the X-ray intensity superimposed.

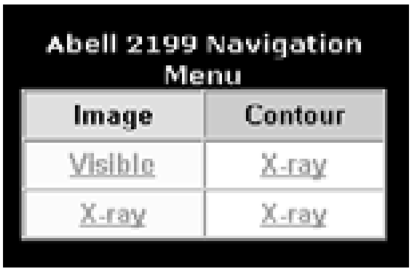

Figure 4 The navigation menu.

- Select Abell 2199 using the cluster selection buttons and then click on X-ray in the left-hand column of the navigation menu. The X-ray image of Abell 2199 will be displayed.
- By clicking on Visible and X-ray in the left-hand column you can flip between the two images to compare the appearance of the cluster in the two different wavebands.
- By clicking the links in the right-hand (Contour) column, either the optical or the X-ray image of each cluster can be displayed with a contour plot of the X-ray emission overlaid. This will help you to compare the regions of emission.
- Now select each of the clusters in turn and compare the optical and X-ray images of all five clusters.

Question 3

Describe in a sentence or two the general similarities or differences between the X-ray images and the optical images. Is there any correspondence between the areas of X-ray emission and the positions of the individual galaxies? Is it likely that the galaxies are the source of the X-ray emission?

Question 4

What is the origin of the X-rays from the cluster, and what does this imply about the structure of the clusters as a whole? (You may need to refer to Chapter 4 of *An Introduction to Galaxies and Cosmology* to answer this question.)

Each of the images in this activity covers a very small field of view (approximately 1 degree square). Now imagine that, instead of these five images centred on known clusters, you are planning to carry out a wide-field survey of a large area of sky in order to find clusters of galaxies.

Question 5

What would be the advantage of conducting a survey to detect clusters from their X-ray rather than their optical emission?

Although the answer to Question 5 might suggest that it should be easy to detect clusters from their X-ray emission – the sensitivity of X-ray telescopes limits the application of this technique. However, one example of such a survey is the Northern ROSAT All-Sky Galaxy Cluster Survey (NORAS), which was carried out using the same PSPC instrument used to take the X-ray images in this activity.

Another problem that has to be overcome in X-ray surveys is that clusters of galaxies are not the only X-ray sources in the sky.

Question 6

What other objects might also appear as bright X-ray sources? How would you exclude these sources from an X-ray survey? (*Hint*: think about the areas of the sky that should be covered in such a survey and the behaviour of other X-ray emitting sources.)

Answers to questions

Question 1

The formula $l = d \times \theta$ (where d is the distance to the cluster, θ is the angular width of the image in radians, and l is the width of the image in Mpc) can be used to calculate the width of each image. For example, the width of the field of view of Abell 2199 is l = 180 Mpc × 0.75° × (1/57.3) = 2.4 Mpc.

The completed Table 2 is given in Table 3.

Table 3 Fields of view for the clusters of galaxies in the image set.

	Abell 2199	Centaurus	Coma	Hydra I	Perseus
distance/Mpc	180	52	100	60	90
angular width of image/degree	0.75	1.0	1.0	1.0	1.5
width of the field of view/Mpc	2.4	0.9	1.7	1.0	2.4

The Abell radius R_A is 2 Mpc. A typical cluster would therefore have a diameter of 4 Mpc. It follows that only the central region of the clusters is shown.

Question 2

Suitable descriptions of the images would be as follows:

Abell 2199 – This is a relatively wide-field image of a distant cluster. There is one large central galaxy with numerous smaller galaxies quite widely spaced. Some of the smaller galaxies appear to be spirals.

Centaurus – This is a relatively narrow field of view of a nearby cluster. There is one large elliptical galaxy at the centre with another large elliptical to the left of centre. The surrounding galaxies appear to contain both ellipticals and spirals.

Coma – This cluster is at intermediate distances compared to the other clusters in the sample. The field of view is relatively wide. There are two large ellipticals surrounded by smaller galaxies. It appears to be a little more tightly packed than A2199. Again some of the smaller galaxies appear flattened. (Although it is not possible to determine morphological types from these images – they are, in fact, mainly highly flattened elliptical galaxies rather than spirals.)

Hydra I – As in the case of Centaurus, this is a relatively narrow field of view of a nearby cluster. The cluster has a tight nucleus of several large galaxies, including one large spiral and a number of other spirals present.

Perseus – The field of view and distance are comparable to the image of the Coma cluster. There are two large central elliptical galaxies and the surrounding galaxies appear to be widely separated. The surrounding galaxies appear to be mainly ellipticals with no obvious spiral galaxies.

Question 3

In each case the area of X-ray emission is much larger than the individual galaxies that are visible in the optical images. The X-ray emission area forms a broad, roughly elliptical envelope surrounding the central galaxies in the cluster. The intensity of X-ray emission is strongest at the centre and falls away smoothly, so that the emission region does not have a sharply defined edge. The X-ray emission does not correspond to individual galaxies but instead appears centred on the centre of mass of the whole cluster. In each case, the X-ray emission region is roughly symmetrical and follows the overall distribution of galaxies, being densest near the centre of the cluster. From these images it seems very unlikely that the galaxies are the source of the X-ray emission.

Question 4

Most of the X-ray emission comes from the space in between the galaxies. The X-rays are not produced within the galaxies, but are the result of *thermal bremsstrahlung* from the intracluster medium (ICM). Since the X-ray emission comes from such an extended region, we can see that the ICM fills the entire volume of the cluster, forming a cloud within which individual galaxies are embedded. The densest and hottest part of the ICM is at the centre. (The process of thermal bremsstrahlung is described in Box 4.1 of *An Introduction to Galaxies and Cosmology*.)

Question 5

In an X-ray image, a cluster would appear as an extended source of X-ray emission against a darker background, and would be easy to see. In an optical image there are large numbers of field galaxies to contend with: to identify a cluster you would need to filter these out and then look for higher than usual concentrations of galaxies. The individual galaxies are also much smaller than the X-ray emission regions, which makes identifying and counting them a more difficult task. The likely locations of clusters would therefore be much more easily identified on a wide-field X-ray image.

Question 6

The other X-ray sources that may be detected can be classified as being either within our Galaxy (Galactic) or external to our Galaxy (extragalactic). The brightest galactic sources would be supernova remnants and X-ray binaries. However, these are objects that tend to be located close to the Galactic plane (see Figure 9.16 of *An Introduction to the Sun and Stars*), and so by carrying out a survey at high Galactic latitudes – away from the Galactic plane – it is possible to avoid most of these sources. The most prominent extragalactic X-ray sources would be AGN, although the X-ray emission from some nearby normal galaxies might also be detected. Extragalactic sources are less easy to exclude than sources within the Galaxy – but the fact that nearby clusters should be extended sources rather than point-like, means that they should be easy to recognize. More distant clusters, whose angular size is below the resolution of the X-ray telescope are more difficult to identify. However, if such an X-ray source was observed to be variable, then that could not be a cluster – since cluster X-ray emission does not change on short timescales (such a source is likely to be an AGN). Ultimately, any unresolved and unvarying X-ray sources would have to be identified by taking an optical spectrum. The presence of strong emission optical lines would clearly reveal the presence of an AGN.

Acknowledgements

Figure 2 and 3(right), and DVD (Hydra I, Centaurus) Based on photographic data obtained using The UK Schmidt Telescope. The UK Schmidt Telescope was operated by the Royal Observatory Edinburgh, with funding from the UK Science and Engineering Research Council, until 1988 June, and thereafter by the Anglo-Australian Observatory. Original plate material is copyright © the Royal Observatory Edinburgh and the Anglo-Australian Observatory. The plates were processed into the present compressed digital form with their permission. The Digitized Sky Survey was produced at the Space Telescope Science Institute under US Government grant NAG W-2166.

Figure 3(left) and DVD (Abell 2199, Coma, Perseus) Based on photographic data obtained using Oschin Schmidt Telescope on Palomar Mountain. The Palomar Observatory Sky Survey was funded by the National Geographic Society. The Oschin Shmidt Telescope is operated by the California Institute of Technology and Palomar Observatory. The plates were processed into the present compressed digital format with their permission. The Digitized Sky Survey was produced at the Space Telescope Science Institute (ST ScI) under U.S. Government grant NAG W-2166.

S282 Astronomy

The geometry of the Universe

Study time: 30 minutes

Summary

This activity relates to a video sequence which provides an overview of how the geometric properties of space depend on its intrinsic curvature.

You should have read to the end of Section 5.3.4 of *An Introduction to Galaxies and Cosmology* before watching this video sequence.

Learning outcomes

- Appreciate the possibility that three-dimensional space need not have a Euclidean geometry, but may be curved in a way that is analogous to the curvature of two-dimensional surfaces.
- Understand how simple geometric tests can be used to determine the curvature of a three-dimensional space.

The activity

In Section 5.3.2 of *An Introduction to Galaxies and Cosmology* you were introduced to the idea of curved space. In particular, you saw that space can have a curvature that is zero or is negative or positive, and that these types of spaces have different geometric properties (see Figures 5.13 of *An Introduction to Galaxies and Cosmology*).

This video sequence sets out to explain the three types of geometry to which the Universe might conform. This is done by exploring the geometrical properties of three types of two-dimensional surfaces that have zero, negative and positive curvature – these are *Euclidean*, *spherical* and *hyperbolic* geometries respectively (see Table 1).

Table 1 The types of geometry described in the video sequence and their corresponding curvatures.

Geometry	Curvature
Euclidean (or 'flat')	zero
spherical	positive
hyperbolic	negative

Throughout this sequence you should bear in mind that the term 'spherical geometry' corresponds to that of space with a positive curvature, whereas 'hyperbolic geometry' refers to a space with a negative curvature. The term 'Euclidean space' is used to describe space with zero curvature – this is also referred to in Section 5.3 of *An Introduction to Galaxies and Cosmology* as a 'flat' space.

To view the video sequence:

- Start the S282 Multimedia guide and then click on The geometry of the Universe under the 'Cosmology' folder in the left-hand panel.
- Press the Start button to run the video sequence.

Note that although Russell Stannard concludes by saying that the geometry of the Universe is unknown, observational results do support the notion that the geometry of the Universe is Euclidean. This observational evidence is discussed in detail in Chapter 7 of *An Introduction to Galaxies and Cosmology*.

After you have watched the video sequence, read the summary provided in the 'Notes' below and then attempt the following questions.

Question 1

What is the definition of a 'straight line' on each of the three surfaces shown in the sequence: flat, spherical and saddle?

Question 2

On the surface of a sphere of radius R, the circumference of a circle is always less than $2\pi r$, where r is the radius of the circle drawn *on* the surface. What can you say about the circumference of a circle that has $r = \pi R$? What would be the circumference of the *largest* circle that could be drawn on the surface?

Notes

The list below indicates the approximate clock time at which each new topic begins.

00:00 Euclidean geometry is but one possible candidate for the geometry of the Universe.

00:15 *Euclidean (flat) geometry* is characterized by:

(i) parallel straight lines which, on being extended to infinity, remain at the same separation from each other

(ii) angles of triangles adding up to 180°

(iii) circles with a circumference $2\pi r$, where r is the radius.

01:20 *Spherical geometry* (positive curvature) is characterized by:

(i) parallel straight lines which intersect after a finite distance

(ii) angles of triangles adding up to >180°

(iii) circles with a circumference $<2\pi r$.

05:00 *Hyperbolic geometry* (negative curvature) is characterized by:

(i) parallel straight lines which progressively diverge

(ii) angles of triangles adding up to <180°

(iii) circles with a circumference $>2\pi r$.

The latter two geometries approximate to flat geometry if the geometrical figures drawn are small compared to the scale of curvature of the surface.

08:20 It was pointed out that it is not immediately obvious what kind of geometry describes the Universe and all that is in it. Measurements with triangles and circles have not been done on a scale large enough to detect which of the three geometries applies to the Universe and are, in any case, impractical.

Video credits

Presenter – Russell Stannard (The Open University)

Producer – Tony Jolly (BBC)

Answers and comments

Question 1

In all three cases the definition of a straight line is in terms of it being the line on the surface that has the shortest distance between two points.

Question 2

If we imagine the circle being centred at the North Pole, then a radius r of length πR would extend to the South Pole. The circle would thus be drawn around the South Pole, and would have a circumference of zero.

As for the largest circle that could be drawn on the surface, that would be the Equator, and hence would have a circumference $C = 2\pi R$.

(Note that this circumference is *not* the Euclidean value of $2\pi r$. In this particular case, the radius r as drawn on the surface is $r = \pi R/2$. This expression can be rearranged to give $R = 2r/\pi$, so the circumference is

$$C = 2\pi R = 4r$$

and this is clearly less than the circumference in Euclidean space, which is given by $C = 2\pi r$.)

S282 Astronomy

The expanding Universe

Study time: 15 minutes

Summary

This activity relates to a video sequence which examines a key concept in general relativistic cosmology – the notion of expanding space.

You should have read to the end of Section 5.4.1 of *An Introduction to Galaxies and Cosmology* before watching this video sequence.

Learning outcomes

- Appreciate the distinction between the expansion of space and movement through space.
- Understand the origin of the cosmological redshift in terms of the expansion of space.

The activity

This video sequence describes the distinction between expanding space and motion through space and explains the origin of cosmological redshift.

There are two points that you need to be aware of prior to watching the video clip. The first is that during the video sequence, Russell Stannard uses the term *horizon*. He refers to 'a galaxy that is so far away that it is beyond the horizon of the observable Universe'. You will meet the term horizon in its cosmological sense in Chapter 6, but a brief explanation is that at any given time in cosmic history, the *horizon distance* is the maximum distance that could be travelled by a physical signal. No physical signal can travel faster than the speed of light, so the horizon distance at any time since the start of cosmological expansion ($t = 0$) corresponds to the distance that light can travel in that time. Clearly, we cannot have received any physical signal from a part of the Universe that lies beyond the current horizon distance.

A second point to note is historical. This video sequence was made in 1994, when the uncertainties on the determination of the Hubble constant were substantially larger than they are at present. So Russell's final comment about not knowing the value of the Hubble constant has been superseded by advances such as a Hubble Space Telescope Key Project which you will meet in Chapter 7.

To view the sequence:

- Start the S282 Multimedia guide and then click on The expanding Universe under the 'Cosmology' folder in the left-hand panel.
- Press the Start button to run the video sequence.

After you have watched the video sequence, read the summary provided in the 'Notes' below and then attempt the following questions.

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Question 1

In what ways was the stretching rubber sheet model a good analogy for the behaviour of space, and in what ways was it inappropriate?

Question 2

How does the cosmological redshift differ from the normal Doppler shift?

Notes

The list below indicates the approximate clock time at which each new topic begins.

00:00 A distinction was drawn between two types of motion – one in which the object moves *through* space, the other where the object is borne along by an *expanding space.* Galaxies take part in both kinds of motion. Only the first is subject to Einstein's special theory of relativity, and therefore cannot exceed the speed of light. With the second type of motion, speeds of distant galaxies might well exceed the speed of light.

03:50 With regard to the cosmological redshift, it was pointed out that the increase in wavelength took place during the journey from the distant galaxy to ourselves. This was due to the fact that the space between the galaxy and ourselves expands, and this stretches out the distance between successive humps (peaks) and troughs in the wavetrain. The greater the distance travelled, the greater will be the stretching and hence the greater the redshift.

Video credits

Presenter – Russell Stannard (The Open University)

Producer – Tony Jolly (BBC)

Answers and comments

Question 1

It was a good analogy in that:

(i) it allowed a demonstration of the two types of motion

(ii) the size of the bound objects (discs/galaxies) did not change as the size of the rubber 'space' between them changed.

The analogy was inappropriate in that:

(i) the rubber belt had an edge to it, whereas three-dimensional space is thought not to have a boundary

(ii) the belt had a central point, whereas all points in three-dimensional space are on the same footing – there is no preferential central point.

Question 2

The normal Doppler shift arises when the humps (peaks) and troughs of a wavetrain have an increased separation during *emission* as a result of the movement of the source relative to the observer. This extended wavelength is then the one that travels, unchanging, across space, and is the wavelength that is subsequently received.

With the cosmological redshift, on the other hand, the wavelength during emission is normal; the extension occurs progressively throughout the subsequent journey.

S282 Astronomy

Redshifts, distances and lookback times

Study time: 45 minutes

Summary

In this activity you will use a piece of software called the *Cosmological modeller* in order to investigate the relationships between distance and redshift, and between lookback time and redshift, in a variety of cosmological models.

You should have read to the end of Chapter 5 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Understand the Hubble expansion of the Universe.
- Understand the concept of mathematical modelling.
- Understand the process of testing a model by comparing predictions of the model with observed results.
- Understand the influence of certain cosmological parameters on the evolution and development of the Universe.

Background to the activity

In Chapter 5 of *An Introduction to Galaxies and Cosmology* you learned about the concept of modelling the Universe, using mathematical models based on Einstein's general theory of relativity and on principles derived from observation, such as the homogeneity and isotropy of the Universe and the Hubble expansion. Typically, such a model will involve a number of cosmological parameters such as the Hubble constant H_0 and the densities of matter and of dark energy.

You were introduced to one such class of models known as the Friedmann–Robertson–Walker (FRW) models. These are based on the Friedmann equation which determines how the scale factor of a model universe evolves over time, and describe a universe that is both homogeneous and isotropic. Although the basic equations of the FRW model do not change, the input parameters can be varied, leading to a class of closely related models. The variations between these models lead to different predictions which can then be compared to observation.

This process of modelling and comparison with observations allows us to determine the values of the input parameters that correspond most closely to those in the actual Universe. As you will learn in Chapter 7 of *An Introduction to Galaxies and Cosmology*, our present best estimate of the values of these parameters comes from the measurements made by the Wilkinson Microwave Anisotropy Probe (WMAP). The FRW model which adopts these parameter values is referred to in this activity as the 'WMAP model'.

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The *Cosmological modeller* program used in this activity consists of two sections: the first uses the FRW class of models to calculate the relationship between the redshift of a source and a number of important properties such as lookback time (defined below), recession speed and distance. It is this part of the program that we shall use in this activity. (The second section, which calculates power spectra of the CMB, is used in a later activity.)

Lookback time is defined as follows: suppose that the light from an object (with redshift z) is observed today, i.e. at time t_{obs}. The light from this object has been travelling through the expanding Universe since the time that it was emitted, t_{em}. The lookback time is defined as the difference between the times of emission and observation ($t_{obs} - t_{em}$).

The activity

1 Introduction

This activity concentrates on the relationships between redshift and lookback times, recession speeds and distances. In particular, we shall try to understand some of these relationships as they apply to the WMAP model of the Universe and to the 'critical model' that you met in Chapter 5.

■ What are the values of the curvature parameter k and the cosmological constant Λ in the critical model?

❑ In the critical model, $k = 0$ (i.e. the model is spatially flat) and the cosmological constant is zero ($\Lambda = 0$).

Let's think about observing a distant galaxy. It seems natural to ask how far away it is, or how long the light from it has been travelling, or even how fast it is receding from us. However, none of these quantities can be measured directly. The one quantity that can actually be measured (from the object's spectrum) is the redshift z. In this activity, then, all of these other quantities are calculated as a function of the redshift. The exact form of each relationship will depend on the parameters used to specify the model.

■ When describing distant objects, why is it preferable to refer to their redshifts as opposed to their distances?

❑ Since the redshift of an object is a *measured* quantity, it is independent of any mathematical model. The distance is an *implied* quantity whose value will depend on the model used. Any mention of distance should really be qualified by specifying the model. It is safer and less ambiguous to refer to the redshift, since it is a quantity that is independent of the assumed cosmological model.

2 Using the Cosmological modeller program

To start the program:

- Start the S282 Multimedia guide, and open the folder called 'Cosmology', then click on the icon for Redshifts, distances and lookback times
- Click Start to open the *Cosmological modeller* program.
- Click on the Redshifts, distances and lookback times button to open the lookback calculator section of the program.

Before starting to use the program, note that the density parameters that the program works with are all the present-day values of these parameters. So strictly speaking, these parameters should all be written in the form that explicitly shows that they relate to the present time, such as $\Omega_{\mathrm{m}}(t_0)$ or $\Omega_{\mathrm{m},0}$. For brevity the program (and these notes) displays the density parameters without the time t_0 (or the subscript 0). However, as you work thought this activity it should be borne in mind that it is the present-day values of these parameters that are referred to.

The lookback calculation screen is divided into a number of sections.

In the lower left-hand corner is a box where you can enter values for the three parameters Ω_{m}, Ω_Λ and H_0 (see Figure 1).

When you first start the program, the values are set as follows:

$\Omega_{\mathrm{m}} = 0.27$

$\Omega_\Lambda = 0.73$

$H_0 = 72\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$

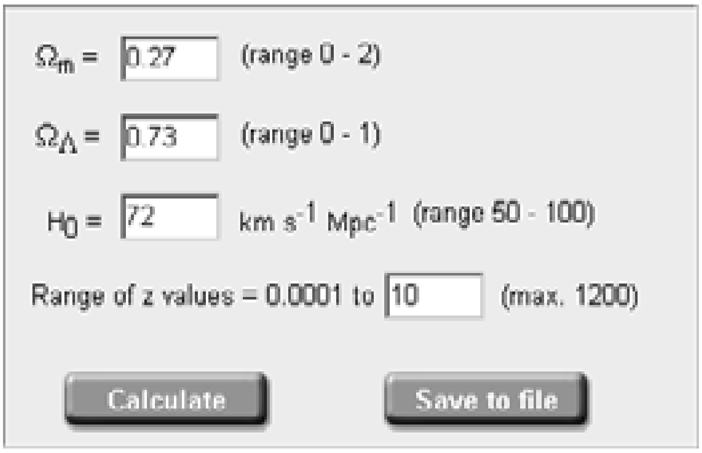


Figure 1 The lookback calculator.

As will be explained in Section 7.5.4 of *An Introduction to Galaxies and Cosmology*, these values represent the best fit with both the WMAP observational data, and with the measurements of very distant Type Ia supernovae. Notice that, in this case, $\Omega_{\mathrm{m}} + \Omega_\Lambda = 1.00$.

■ What can be inferred about the curvature of an FRW model in which $\Omega_{\mathrm{m}} + \Omega_\Lambda = 1$?

❑ If $\Omega_{\mathrm{m}} + \Omega_\Lambda = 1$, then the curvature parameter $k = 0$, i.e. the model has zero curvature. (See *An Introduction to Galaxies and Cosmology*, Question 5.9. Note also that it has been assumed that any contribution to the total density from radiation is negligible.)

We will start by investigating the properties of this WMAP model, and you should leave the parameters set to these default values for the exercises described in Sections 3 to 7 below.

3 Age of the Universe

Perhaps the most fundamental prediction that we require of any model is an estimate of the overall age of the Universe. Before making calculations with the WMAP model, let's get a sense of scale by considering for a moment the prediction of the simpler 'critical model'. You may recall that there is a simple relationship between the Hubble constant and the age of a critical model universe. Consequently, as the following question illustrates, we don't need to use the *Cosmological modeller* to carry out this calculation.

Question 1

According to the critical model of the Universe described in Section 5.4.3, the age of the Universe is two-thirds of the Hubble time. Given a value of the Hubble constant of 72 km s^{-1} Mpc^{-1}, calculate the Hubble time ($1/H_0$), and hence the age of a critical model Universe with this value of H_0. Is this age consistent with observations of the ages of stars in globular clusters?

(Note that 1 year = 3.16×10^7 s, and 1 Mpc = 3.09×10^{22} m.)

You should now compare this result with the prediction of the WMAP model. To do this, you will need to use the *Cosmological modeller*.

- Leaving the model parameters set to their default values as shown above, press the Calculate button.

A graph will appear on the right-hand side of the screen. However, for the moment we are interested in the calculated value for the age of the Universe. This is displayed in the area to the right of the data entry box, below the graph.

- How does the age of the Universe calculated by the program compare with the Hubble time and with the age of the critical model universe calculated in Question 1?

- The value calculated *for this model* by the *Cosmological modeller* (i.e. as specified by the combination of input parameters) is very close to the Hubble time – about 13.5 billion years. This is substantially greater than the age of the critical model (which is 2/3 of the Hubble time).

So, the age of the WMAP model is 50% greater than the age of the critical model (assuming a value of H_0 = 72 km s^{-1} Mpc^{-1}). Up until the early 1990s, the critical model was the model of choice for most astronomers and cosmologists. However, the relatively short age of the critical model universe was found to be inconsistent with the ages of the oldest stars in the Universe. The greater age (for a given value of H_0) of an accelerating model such as the WMAP model, removes this inconsistency.

4 Lookback time as function of redshift

Now take a look at the graph displayed at the upper right portion of the screen. This should be a plot of lookback time against redshift. If not, click on the arrow in the Choose graph box at the top left and select the first option: Lookback time v z (see Figure 2).

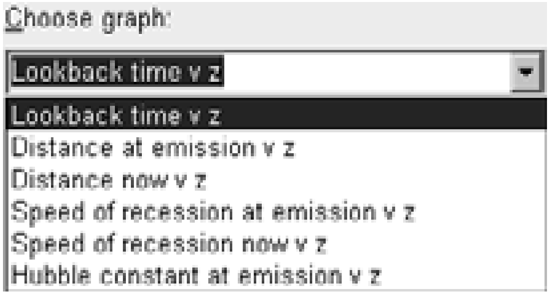


Figure 2 Selecting an option in the Choose graph box.

This graph shows the relationship between redshift z and *lookback time* (in billions of years).

Question 2

You may recall from Section 5.4.1 of *An Introduction to Galaxies and Cosmology* that cosmological redshifts result from the expansion of space between the time of emission of light from a source and the time that it is received on the Earth. Consider the point on the graph with $z = 5.0$. By what amount has the scale factor of the Universe increased since the light was emitted?

In this model, $z = 5.0$ corresponds to a lookback time of 12.3 billion years. The light has been travelling for this length of time through an expanding Universe, its redshift increasing as the scale factor changes, until its detection in the present day.

This is an important point to bear in mind as you look at the other graphs – the redshift depends on the time at which the light was emitted. Specifically, as the answer to Question 2 shows, redshift is related to the scale factor of the Universe at the time of emission.

- Higher redshifts correspond to earlier times of emission.
- A higher redshift implies a smaller value of the scale factor at the time of emission.

Now consider the overall shape of the lookback time graph. The graph is not a straight line, but rises steeply and then flattens off. At high redshifts ($z > 10$), the lookback time approaches the total age of the Universe.

Question 3

Based on your answer to Question 2, write a brief explanation of the main features of the lookback time graph. You should comment on the following features:

(a) The overall shape of the curve – the lookback time increases as redshift increases.

(b) The fact that the curve is not a straight line, rising steeply at low redshifts but flatttening out at high redshifts.

(It isn't necessary to explain the exact numbers – concentrate on accounting for the overall shape and features of the graph.)

The exact shape of the curve will of course depend on the amount of acceleration or deceleration of expansion and you will explore this in more detail in Section 8.

5 Distance as function of redshift

After lookback time, the next parameter to consider is the distance to a source as a function of its redshift. You can display these plots for the current model simply by selecting from the Choose graph box – there is no need to recalculate unless you change one of the model parameters. Note that there are two distance options in the Choose graph box: Distance now and Distance at emission.

■ Why is it necessary to have these two different plots of distance against redshift?

❑ As seen from the plot of lookback time against redshift, the light emitted from distant galaxies has travelled for a considerable length of time before being received on the Earth. During the time that the light has been travelling, the Universe has continued to expand, so the present-day distance will be much greater than the distance to the source when the light was emitted – more so for higher values of z.

Take a look at the Distance now plot first. As with the lookback time, it is probably not surprising that objects whose light has been redshifted by a greater amount lie at further distances from the Earth. As with lookback time, the distance is not directly proportional to redshift, but flattens off at higher redshifts.

Now switch to the Distance at emission plot. You might initially be surprised to see that the curve peaks between $z = 1$ and $z = 2$, and that the emission distances for larger redshifts start becoming smaller again! Take a moment to think about why this might be, before attempting Question 4 (remember that light received today from objects with different values of z has been travelling for different lengths of time).

Question 4

To understand the relationship between the two plots, complete Table 1 by filling in the values of $(z + 1)$, Distance now and Distance at emission for each of these specific values of redshift. You should be able to deduce a simple relationship between the two curves and the value of $(z + 1)$.

(In order to complete the table you can simply read the values from the graph, using the Scale graph slider to change the scale if necessary. Alternatively, you can use the Save to file button to create a spreadsheet file containing exact values. Note that the file will contain the data for all graphs that can be plotted. This file can be opened with StarOffice.)

Table 1 Grid for recording results for distance as a function of redshift.

z	$z + 1$	**Distance now**	**Distance at emission**
1.0			
3.0			
5.0			
7.0			
9.0			

Again, the precise shape of the distance–redshift curves will depend on the parameters of the model used, and you will investigate this further in Section 8.

6 Recession speed as function of redshift

Now select the plot Speed of recession now v z. You should see that, as expected, the speed of recession increases as a function of redshift: objects whose light is more redshifted are receding more rapidly from the Earth.

Up to now, you have probably left the Range of z values option set to 10. Before attempting Question 5, you may find it helpful to zoom in a little.

- Change this value to 5 and press the Calculate button before proceeding.

Question 5

As with the distance plots, there is a second plot: Speed of recession at emission v z. Compare the two plots by switching between them. Has the speed of recession for a given redshift increased or decreased between the time the light was emitted and the present time? What can you deduce about the rate of expansion of the Universe in this particular model – is it accelerating or decelerating?

■ You may also have noticed that for redshifts greater than about 2.0, the speeds of recession are greater than the speed of light! How is this possible, and how can light reach us from a galaxy with a recession speed greater than c?

❑ These speeds are the result of the expansion of space, and not of the movement of the galaxies *through* space.

They might more properly be thought of as a rate of increase of separation, rather than a recession speed. You may be aware that special relativity tells us that the speed of light is constant in any frame of reference: light emitted form a receding source does not travel slower than light emitted from a stationary source. Once the light has been emitted from an object, it always travels at a speed c relative to the *local* space, not relative to the source.

7 Hubble parameter as function of redshift

The next graph to consider is that of Hubble parameter against redshift. (Note that the cosmological modeller refers to the time varying Hubble parameter as the 'Hubble constant'.) In order to get a perspective on what to expect, let's first think about what would happen in a Universe with no acceleration or deceleration of the expansion rate.

If the rate of expansion were constant, the scale factor $R(t)$ would increase at a constant rate. So the rate of change of the scale factor $\dot{R}(t)$ would be constant.

■ In such a Universe what would happen to the Hubble parameter over time?

❑ The Hubble parameter is defined in Equation 5.21 as the rate of change of the scale factor divided by the scale factor.

$$H(t) = \frac{\dot{R}(t)}{R(t)}$$

Since $\dot{R}(t)$ in this scenario is constant, this implies that the Hubble constant would *decrease* over time as the scale factor increases according to

$$H(t) \propto 1/R(t) \qquad \text{(A)}$$

However, the redshift of an object and the scale factor $R(t)$ at the time of emission are related (see the answer to Question 2)

$$R(t) \propto \frac{1}{1+z} \qquad \text{(B)}$$

So, combining Equations A and B

$$H(z) \propto (1 + z)$$

Note that we have now written H as a function of z rather than t to emphasize that we are interested in the way in which H varies with redshift rather than time.

You may wish to try sketching the graph of Hubble parameter against redshift for this situation. You should get a straight line crossing the vertical axis ($z = 0$) at its present-day value of 72 km s^{-1} Mpc^{-1}.

Question 6

In a universe with a constant rate of change of scale factor ($\dot{R}$ = constant) and a present-day Hubble constant of 72 km s^{-1} Mpc^{-1}, a distant galaxy is observed to have a redshift of $z = 10$. What was the value of the Hubble parameter at the time of emission in this model?

We have just considered how the Hubble parameter varies with redshift z in a universe which has a constant expansion rate. Now we are in a position to compare the behaviour of this type of universe with that of the WMAP model.

- Select Hubble constant at emission v z from the Choose graph box at the top left.
- (If not already selected) select a range of z from 0 to 5, and examine the graph carefully.

In light of what you have observed, consider the following.

- Question 6 showed that, for a constant rate of expansion, the Hubble parameter would be proportional to $(z + 1)$. If, for a given z, a model predicts a higher value for the Hubble parameter than this constant expansion model, does that imply that the rate of expansion was *greater* or *smaller* in the past than its present value?

❑ A given z corresponds to a particular value of scale factor at the time of emission. If the value of $H(z)$ at this time is higher than that predicted by the constant expansion model, this implies that the rate of expansion in the past was greater than its present value.

So if a model predicts *higher* values of $H(z)$ than the constant expansion model, this implies that the expansion has *decelerated* since the light was emitted.

Conversely, if a model gave a value of $H(z)$ that was *less* than that predicted by the constant expansion model, that would imply an *acceleration* in the rate of expansion.

Question 7

By filling in Table 2, compare the Hubble parameters predicted by the WMAP model with what you would expect from constant $\dot{R}$ model described in Question 6. What conclusions can you draw about the rate of expansion of the Universe in the WMAP model? (Again you can estimate the readings from the graph, or save the values to a spreadsheet file.)

Table 2 Grid for recording Hubble parameters predicted with different models.

z	$H(z)$/km s^{-1} Mpc^{-1}	
	constant $\dot{R}(t)$ model	**WMAP model**
0	72	
1		
2		
3		
4		
5		

8 Comparison with the critical model

Finally, let's compare the results that you have obtained with the predictions of another model mentioned in Chapter 5 – the matter-dominated critical model, which has $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$.

- ■ This critical model has no contribution from the cosmological constant Λ. What are the main differences that you would expect between the models?
- ❑ With a critical density of matter and no cosmological constant, the expansion of this Universe will continue to decelerate. The age of this Universe will be approximately 2/3 of the Hubble time and the evidence for recent accelerated expansion seen in Sections 6 and 7 should be absent from this model.

Now let's investigate the model in more detail using the cosmological calculator.

- ■ Set $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$ and press the Calculate button.

Question 8

Repeat each of the plots that you made for the WMAP model. For each plot, note the main similarities and differences between the two models, and summarise these differences under the following headings: Lookback time, Distance, Speed and Hubble parameter.

Feel free to experiment with other combinations of parameters – you will need to keep within the constraints of a flat Universe ($(\Omega_m + \Omega_\Lambda) = 1$). Each time, make the same comparisons as you did in Question 8 and note the changes in behaviour caused by the changes in parameters.

Conclusions

During the course of this activity you have investigated how a number of cosmological parameters behave as a function of redshift. The redshift of a given object depends on the amount that the scale factor of the Universe has changed since the light observed today was emitted from the object. In all cases, the redshift is the only directly observable property and the calculated values of lookback time, distance, recession speed, and Hubble parameter all depend on the details of the cosmological model chosen.

Comparison between two different cosmological models has given differing predictions for the evolution of the Universe. The critical model predicts a consistently decelerating Universe and gives some predictions (such as the age of the Universe) that are at odds with observation.

By contrast an FRW model using values obtained from the WMAP measurements predicts a history of the Universe involving a decelerating expansion at early times and a more recent phase in which the expansion is accelerating. As you will see in Chapter 7, this description is also consistent with independent measurements of Type Ia supernovae and with the ages of stars in globular clusters. Hence this WMAP model is to be preferred over the critical model.

Answers to questions

Question 1

The value of H_0 of 72 km s^{-1} Mpc^{-1} must first be converted into units of years and Mpc.

$$H_0 = \frac{72\ \text{km s}^{-1}\ \text{Mpc}^{-1} \times 3.16 \times 10^7\ \text{s yr}^{-1}}{3.09 \times 10^{19}\ \text{km Mpc}^{-1}} \quad \text{[units: yr}^{-1}\text{]}$$

So $$\frac{1}{H_0} = \frac{3.09 \times 10^{19}\ \text{km Mpc}^{-1}}{72\ \text{km s}^{-1}\ \text{Mpc}^{-1} \times 3.16 \times 10^7\ \text{s yr}^{-1}} \quad \text{[units: yr]}$$

giving a *Hubble time* of 1.36×10^{10} years. However, in the critical model the age of the Universe is 2/3 of the Hubble time, i.e. about 9×10^9 years. This is less than the ages of stars in some globular clusters and is evidence that the critical model cannot be correct.

Question 2

The redshift is related to the scale factor by Equation 5.13 of *An Introduction to Galaxies and Cosmology*:

$$z = \frac{R(t_{\text{obs}})}{R(t_{\text{em}})} - 1$$

So, rearranging

$$\frac{R(t_{\text{obs}})}{R(t_{\text{em}})} = z + 1$$

$$\frac{R(t_{\text{em}})}{R(t_{\text{obs}})} = \frac{1}{z + 1}$$

So, at a redshift of $z = 5$, which in this model corresponds to a lookback time of 12.3 billion years,

$$\frac{R(t_{em})}{R(t_{obs})} = \frac{1}{5+1} = \frac{1}{6}$$

The scale factor when the light was emitted $R(t_{em})$ must have been one-sixth of its present-day value. The scale factor has therefore increased by a factor of six since the time of emission.

Question 3

(a) The first feature to note is that the lookback time rises continuously as a function of redshift. This is to be expected in a Universe that is expanding continuously. Higher redshifts correspond to smaller scale factors at the time of emission. Provided the scale factor is always increasing as a function of time, then – even if the rate of expansion changes – higher redshifts must correspond to earlier times of emission and hence greater lookback times.

(b) The other feature of the graph is that it rises rapidly at low redshifts and flattens off as the redshift increases. This can be understood by thinking about Equation 5.13 of *An Introduction to Galaxies and Cosmology*. A redshift of 1.0 means that the scale factor of the Universe was one-half its present value at the time of emission. From this point, it has taken 7.6 billion years for the scale factor to double in size to its present value. By contrast, the time taken for the scale factor to increase from one-eleventh to one-tenth of its present size would have been very much less than this, hence there is only a small difference in lookback times between $z = 9$ and $z = 10$.

Question 4

The completed Table 1 should look like Table 3 (note that the table shows values for distance in billions of light years and in Gpc since the *Cosmological modeller* allows you to choose your preferred unit of distance measurement).

Table 3 Results for distance as a function of redshift.

z	$z + 1$	Distance now/billion light-years*	Distance at emission/billion light-years*
1.0	2.0	10.7 (3.27)	5.33 (1.64)
3.0	4.0	20.8 (6.37)	5.20 (1.59)
5.0	6.0	25.5 (7.83)	4.25 (1.30)
7.0	8.0	28.4 (8.70)	3.54 (1.09)
9.0	10.0	30.3 (9.30)	3.03 (0.93)

*The table shows values for distance in billions of light-years and in Gpc (given in brackets).

You should be able to see that: (Distance now) = (Distance at emission) × (z + 1).

Since (z + 1) is simply a measure of how much the Universe has expanded since the light was emitted, it makes sense that the distance at emission is smaller than the distance now by that factor.

Another way of looking at it is to re-arrange Equation 5.13:

$$(z+1) = \frac{R(t_{\text{obs}})}{R(t_{\text{em}})}$$

It is important to realize that distance does increase with redshift provided that all measurements are made *at the same time*.

However, points on the distance at emission plot do *not* correspond to measurements made at the same time. Higher z values correspond to earlier emission times. For z values greater than 2 the smaller scale factor at time of emission becomes the dominant factor, making the emission distances decrease.

Question 5

You should have noticed that there is a difference in the behaviour of the curves at high redshifts and low redshifts. Above $z = 2.5$, the speed of recession today for a given object is *less* than it was at the time of emission, implying that the expansion of the Universe is slower than it was in the past. For lower redshifts $z < 2.5$, the opposite is true: the present-day speed is higher than the speed at emission, implying that the expansion is accelerating.

This apparent contradiction can be resolved by thinking about the *times of emission*. All the models discussed in Section 5.4.2 predict a decelerating expansion during the early stages of the Universe. Only at later times does the density of matter decrease sufficiently for the cosmological constant to cause the expansion to start accelerating. Thus more recent events (low z) show evidence of the acceleration, whereas at earlier times ($z > 2.5$) the Universe was still decelerating.

Question 6

In a model universe in which $\dot{R}$ is constant, the Hubble parameter is proportional to $(1 + z)$,

$$H(z) \propto (1 + z)$$

So using the fact that $H(z = 0) = H_0$, we can write

$$H(z) = H_0 \times (1 + z)$$

Using $H_0 = 72\ \text{km s}^{-1}\ \text{Mpc}^{-1}$, if $z = 10$

$$H(z = 10) = (72\ \text{km s}^{-1}\ \text{Mpc}^{-1}) \times (10 + 1) = 792\ \text{km s}^{-1}\ \text{Mpc}^{-1}$$

Question 7

The completed Table 2 is given in Table 4. The first column can be filled by using the relationship established in Question 6: that the Hubble parameter is proportional to $(z + 1)$. The second column has been filled using values from a saved spreadsheet of the results from the $\Omega_{\text{m}} = 0.27$, $\Omega_{\Lambda} = 0.73$ model.

Table 4 Hubble parameters predicted with different models.

z	$H(z)$/km s^{-1} Mpc^{-1}	
	constant $\dot{R}(t)$ model	**WMAP model**
0	72	72
1	144	122
2	216	204
3	288	306
4	360	422
5	432	553

There is again a difference in behaviour between the lower and the higher redshift ranges. For $z = 1$ and $z = 2$ the Hubble parameter is less than the constant-expansion prediction, implying an accelerating expansion. For $z = 3$ and greater, the opposite is true.

This is again consistent with the picture of a decelerating early Universe, switching over to an accelerating expansion during more recent times.

Question 8

Lookback time

The curve has the same form, rising rapidly and then flattening out – but in the critical model it reaches a lower maximum value, giving an age of about 9 billion years, or 2/3 of the Hubble time, as expected.

Distance

The distance now and distance at emission curves are still related by $(z + 1)$, but in the critical model the distances are always smaller than in the WMAP model. Again, this is to be expected since the expansion in the critical model is decelerating.

(You will see in Chapter 7 of *An Introduction to Galaxies and Cosmology* that one of the pieces of evidence for a non-zero cosmological constant is that the Type Ia supernova measurements found that the distances for a given z were *greater* than would be expected in the critical model. The WMAP model is therefore consistent with this finding.)

Speed

You should find that the two speed graphs behave differently to those in the WMAP model. In the critical model, the speed now is smaller than the speed at emission for all values of z. This indicates that the Universe is still decelerating, with even very recent emission events having lower speeds today than at the time of emission.

Hubble parameter

Again, the Hubble constant vs. z plot does not show any of the evidence of recent acceleration that you saw in Section 7. By $z = 1$, H has already increased to 200 km s^{-1} Mpc^{-1} and the whole curve lies higher than the constant expansion line.

S282 Astronomy

CMB power spectra

Study time: 60 minutes

Summary

In this activity you will use the *Cosmological modeller* software package to predict the power spectrum of the cosmic microwave background (CMB) in a range of cosmological models. This activity will help you appreciate how the values of cosmological parameters can be constrained by measurements of the CMB.

You should have read to the end of Chapter 7 of *An Introduction to Galaxies and Cosmology* before starting this activity.

Learning outcomes

- Understand the concept of mathematical modelling.
- Understand the process of testing a model by comparing predictions of the model with observed results.

Background to the activity

In Section 7.5 of *An Introduction to Galaxies and Cosmology* you saw how intrinsic anisotropies in the cosmic microwave background could be described mathematically in terms of a power spectrum and how this allows a more detailed analysis to be made than by simple visual assessment of anisotropy maps. Specifically, the mathematical description provided by the power spectrum allows the results from observational measurements to be compared *quantitatively* with the predictions of different theoretical models.

The Power spectrum calculator section of the *Cosmological modeller* program used in this activity uses the FRW models, as described in Chapter 7, to simulate the fluctuations in the cosmic microwave background and compare the resulting predicted angular power spectrum with the power spectrum of the WMAP anisotropy map.

In this activity you will explore the sensitivity of the power spectrum to a number of cosmological parameters such as the density of matter and the cosmological constant. In doing so you will get a feel for the way that the most probable values of these parameters can be determined by comparing the power spectra predicted by the models with real observational data such as that from WMAP.

The activity

The *Cosmological modeller* program has a section that predicts the expected variations in the cosmic microwave background for different cosmological models. It does this by calculating and plotting the angular power spectrum of the CMB as described in Section 7.5 of *An Introduction to Galaxies and Cosmology*. Although a brief description of this plot is given here, you may wish to refer to Sections 7.5.2 and 7.5.3 of *An Introduction to Galaxies and Cosmology* whilst working through this activity.

- Start the S282 Multimedia guide, and open the folder called 'Cosmology', then click on the icon for CMB power spectra.
- Click Start to open the *Cosmological modeller* program.
- Click on the CMB power spectra button to open the power spectrum section of the program.

Calculating the power spectrum

The Power spectrum calculator section of the program allows to you explore the effects of changing the various parameters that specify the model and hence compare the predictions of different cosmological models.

The power spectrum calculator has a data entry area where you can enter the parameters that specify the model (Figure 1).

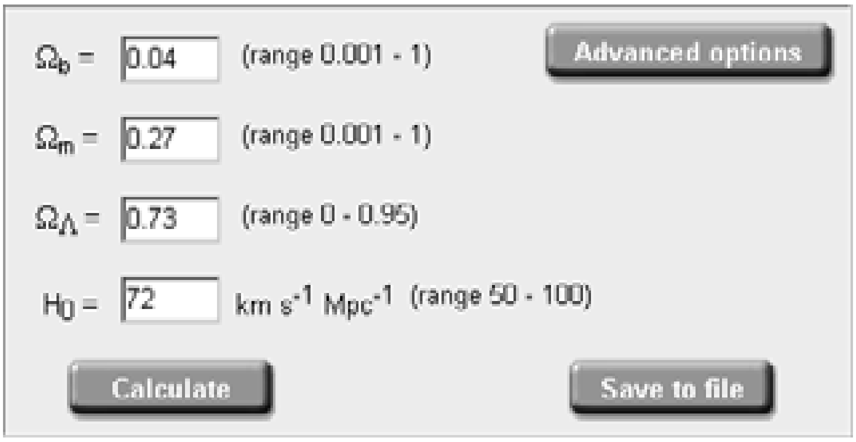


Figure 1 The power spectrum calculator.

Note that the density parameters that the program works with are all the present-day values of these parameters. So strictly speaking, these parameters should all be written in the form that explicitly shows that they relate to the present time, such as $\Omega_b(t_0)$ or $\Omega_{b,0}$. However for brevity the program (and these notes) display the density parameters without the t_0 or the subscript 0. However, as you work thought this activity you should remember that it is the present-day values of these parameters that are referred to.

When the program starts, the model parameters are set to the default values shown in Figure 1. Notice that the input to the program includes the parameter that quantifies the density of baryonic matter Ω_b. Note also that Ω_m represents the total matter density – baryonic plus dark matter – and therefore includes Ω_b. In this model baryonic matter contributes about 15% of the total matter density.

- To start with, leave the parameters set to the values shown above and click on the Calculate button to generate the power spectrum for this model.

The calculation may take a few minutes. Once the calculation is complete, you should obtain a curve similar to that shown in Figure 7.22 of *An Introduction to Galaxies and Cosmology*, with peaks at values of l of about 200, 500 and 800.

When interpreting these power spectra remember that the horizontal axis represents the angular scale of the fluctuations in the CMB (with smaller angular scales to the right), and the vertical scale represents the angular power. The angular power provides a quantitative measure of the size of fluctuations at a given angular scale.

Question 1

The scale on the horizontal axis of this plot runs from angular scales of a few degrees down to about 0.1 degree. Compare the angular scales that correspond to the peaks in this power spectrum with the resolution of COBE all-sky map (*An Introduction to Galaxies and Cosmology*, Figure 7.16) and that of the WMAP all-sky map (*An Introduction to Galaxies and Cosmology*, Figure 7.21). Which of these maps (COBE or WMAP) do you think could show fluctuations that correspond to the peaks is this power spectrum?

In answering Question 1 you should have reached the conclusion that the displayed power spectrum shows variation over scales that were mapped by the WMAP mission. In fact, the default values of the program are close to the values given in Section 7.5.4 of *An Introduction to Galaxies and Cosmology* as the best fit to the WMAP data. You can display the WMAP data and best-fit model to these data by selecting the options (in tick boxes at the upper right-hand part of the screen) MAP data and best fit to data.

In the next section you will compare this prediction with that of a very different model. At any stage through these exercises, you can compare the predictions of a given model with the WMAP data by using the MAP data and best fit to data options.

Comparison with critical model

Prior to the Type Ia supernova measurements described in Section 7.3.2 of *An Introduction to Galaxies and Cosmology*, one of the preferred cosmological models was the matter-dominated critical density model with $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$. In this section of the activity you will look at what this model predicts for the fluctuations in the CMB.

- For the critical Universe model the density parameters are set to $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$. If the proportion of baryonic matter is to remain at 15%, what would be the value of Ω_b required for this model?

- ❑ Since the total matter density Ω_m is 1.0, in order to maintain the 15% proportion of baryonic matter, Ω_b should be set to 0.15.

- Enter the values $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$ together with the appropriate value for Ω_b (= 0.15), and select Calculate again. After some delay, the curve for the critical model should be displayed, superimposed on the previous curve.

Question 2

Looking at the differences between the two curves, describe any similarities and differences between them. How would the corresponding all-sky maps of the CMB differ from one another?

The effect of varying the cosmological constant

Clearly the comparison with the critical density model represents quite a drastic change which would give quite a different pattern to the CMB. We will now consider some more subtle differences between models. First, let's investigate the effect of changing the amount of cosmological constant by varying Ω_Λ.

Because there are a number of parameters that can be varied, it would be very easy to get confused by changing several properties of the model at once. In order to make meaningful comparisons it is necessary to work within certain constraints, so that only one characteristic of the model is changed at any one time. Specifically, to isolate the effect of varying Ω_Λ we want to maintain a flat Universe and to keep the proportion of baryonic matter constant (you will investigate the effect of changing this proportion in the next section).

- As the value of Ω_Λ is varied, what effects do these constraints have on the values of Ω_m and Ω_b?

- ❑ In order to maintain a flat Universe, ($\Omega_m + \Omega_\Lambda$) should add up to 1.0. Thus, if Ω_Λ is changed, Ω_m must also be changed to maintain a total of 1.0.

To keep the proportion of baryonic matter constant Ω_b should be changed at the same time to keep its value at 15% of the value of Ω_m.

- Based on these considerations, select Clear graph and make three plots with the following combinations of parameters, leaving H_0 set to 72 km s^{-1} Mpc^{-1} throughout (don't clear the graph between plots – just enter the parameters and select Calculate, so that you end up with the three curves superimposed).

Model	Ω_b	Ω_m	Ω_Λ
1	0.06	0.4	0.6
2	0.04	0.27	0.73
3	0.03	0.2	0.8

These three sets of parameters represent a small variation of Ω_Λ either side of the best-fit value, keeping the proportion of baryonic matter fixed at 15%.

- How much difference is there between these three plots, compared to the critical model plots? How easy is it to distinguish between these three models?

- ❑ There is very little difference between these three models. Varying the cosmological constant within the constraints given above appears to have had little effect on the prediction for the CMB.

You may wish to print a copy of these plots before proceeding. (Select the Print option from the File menu.)

So varying the parameters in a systematic way has allowed us to isolate the effects of a single property of the model. In this case, varying the proportion of cosmological constant by a small amount either way has relatively little effect on the predicted power spectrum, although, as you saw earlier, eliminating it entirely did produce a large effect. This tells us that a cosmological constant is definitely required in the model. Once Ω_Λ is added, the power spectrum is relatively unchanging and is not very sensitive to small changes either side of the 'best-fit' value of 73%. We will consider implications of this lack of sensitivity to the exact value of Ω_Λ in the conclusion to this activity.

The effect of varying the proportion of baryonic matter

In a similar vein to the previous section, let's now explore the effect of changing the proportion of baryonic matter by varying Ω_b slightly whilst keeping the other parameters fixed.

- Select Clear graph and again make three plots, this time using the following parameter combinations.

Model	Ω_b	Ω_m	Ω_Λ
4	0.02	0.27	0.73
5	0.04	0.27	0.73
6	0.06	0.27	0.73

Question 3

Compare these three plots with the three plots made above when you investigated the effect of varying Ω_Λ. Which parameter (Ω_Λ or Ω_b) has the larger effect on the shape of the power spectrum, and why?

You may want to explore further using different combinations of parameters. Again, try to make changes subject to reasonable constraints and work to a plan in order to generate meaningful comparisons.

Conclusions

During the course of this activity you should have developed a feel for the way in which the power spectrum allows detailed mathematical comparisons of different models to be made. By making small adjustments to the parameters the predictions of the model can be 'fine-tuned' to obtain a best fit to the observational data. In doing so it has been possible to come up with a combination of the parameters Ω_Λ, Ω_m and Ω_b that gives a good agreement with the power spectrum of the WMAP anisotropy map.

The results of such curve-fitting activities should be taken more as an indication of the *probable values* of these parameters rather than as conclusive proof. For one thing, you have seen that the model is more sensitive to the proportion of

baryonic matter than to the exact value of cosmological constant, so is a better predictor of some parameters than of others. Furthermore, it may be that other combinations of parameters may be found that give similar degrees of agreement with the observations. For these reasons, other independent observational data – such as the supernova measurements – should also be used to confirm and place further constraints on the values of the parameters, as discussed in Section 7.5.5 of *An Introduction to Galaxies and Cosmology*.

Using a combination of techniques has now allowed the values of the main cosmological parameters to be determined to a high degree of confidence as detailed in Figure 7.27 of *An Introduction to Galaxies and Cosmology*.

Answers to questions

Question 1

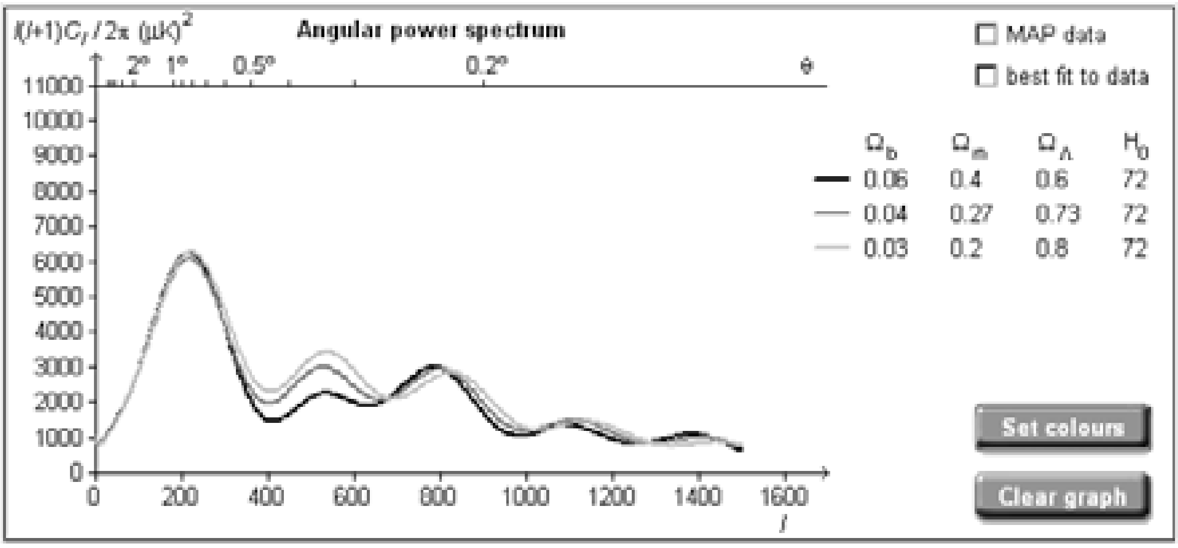
The peaks in the power spectrum at $l = 200$, 500 and 800 correspond to angular scales of about 0.8°, 0.35° and 0.2°, indicating that there is fluctuation on these very small angular scales in the CMB. The COBE map has a much lower angular resolution than this (~7°, see *An Introduction to Galaxies and Cosmology*, Section 7.5.2), so the peaks in this power spectrum would only be seen as fluctuations in the WMAP anisotropy map which has an angular resolution of about 0.1°.

Question 2

Both curves have a peak at around the $l = 200$ mark, so the largest variations in the all-sky maps from these two models would be on similar scales. However, the pattern of the peaks at higher l values is quite different. In particular, there is a large difference between the power spectra at around $l \sim 500$. In the universe that is characterized by the best-fit parameters to the WMAP data, there is a strong peak at $l \sim 500$ and so we would expect to see strong fluctuations on an angular scale of about 0.35°. However, in the critical density model the $l \sim 500$ peak has disappeared, so variations on the corresponding angular scale (0.35°) would be absent from the CMB map in such a universe.

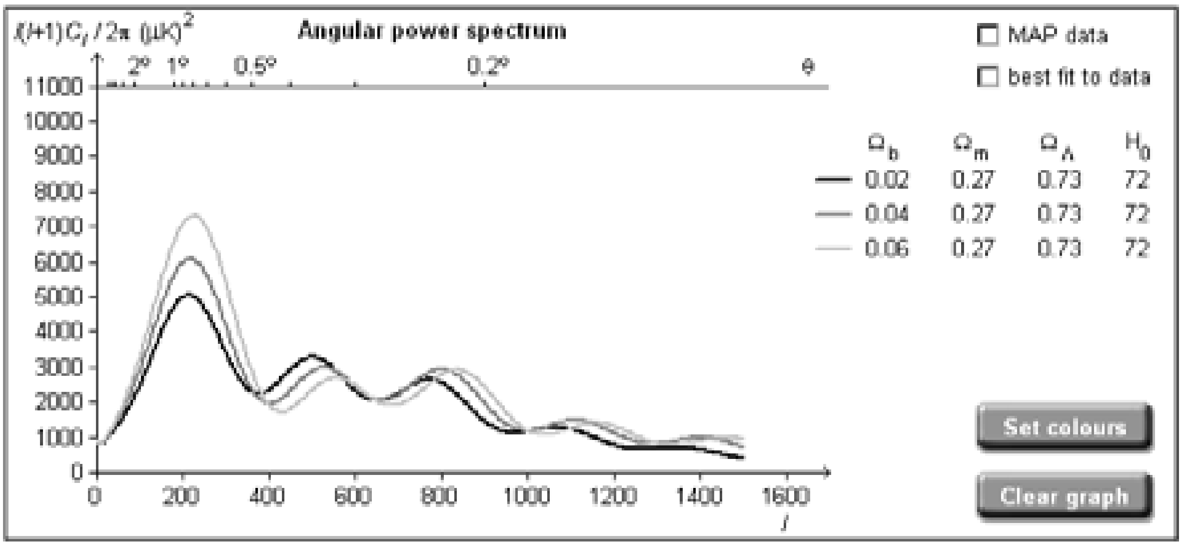
Question 3

You should have obtained the following three plots for the variation of Ω_Λ:



These three curves are all very similar, with peaks in the same locations and only small variations in their amplitudes.

You should have obtained the following three plots for the variation of Ω_b:



These curves show a much larger effect, with both the positions and the amplitudes of the peaks changing. In this comparison, the density of baryonic matter has the greatest effect on the power spectrum of the CMB.

The main process responsible for generating the fluctuations is the interaction of photons with baryonic matter as described in Section 7.5.3 of *An Introduction to Galaxies and Cosmology*. Thus it is hardly surprising that the density parameter of baryonic matter is an important factor in determining the precise shape of the power spectrum.

S282 Astronomy

A history of the Universe

Study time: at least 60 minutes

Summary

This is an open-ended activity based around a multimedia package, which describes the important events in the evolution of the Universe from the very earliest times. The package contains animations, simulations and interviews with prominent astronomers and cosmologists.

The content of this package is of particular relevance to Chapters 6 and 8 of *An Introduction to Galaxies and Cosmology*.

Learning outcomes

- Understand of the significance of key events in the history of the Universe.
- Appreciation of whether the physics of certain processes in the history of the Universe is speculative or well understood.
- Development of skills in locating information relevant to a particular topic.

Background to the activity

This activity is somewhat different to the others that you have encountered in the course in that it is more open-ended. It is based around a multimedia package called ‘A history of the Universe’ which was originally developed for the course S103 *Discovering science*. The package has been updated for S282 and new sections have been added to cover some of the recent changes in cosmology. Even if you have studied S103, it is strongly recommended that you use this package to support your understanding of Chapters 6 and 8 of *An Introduction to Galaxies and Cosmology*.

Before using the package you should be aware of the following points.

Some of the numbers quoted in the package for times, temperatures and energies are slightly different to those quoted in *An Introduction to Galaxies and Cosmology*. In much of cosmology, only order of magnitude estimates can be made about physical conditions, and any differences between the course text and this package are insignificant.

The package has a little more detail about particle physics than is introduced in the main text. For instance, the package mentions hypothetical particles called X-bosons which are not discussed in *An Introduction to Galaxies and Cosmology*. These instances where the package goes beyond the main text should be considered as supplementary information – and not part of the core material that you would be expected to know about in S282.

The activity

The following are some suggestions of how you might use ‘A history of the Universe’ package.

1 Prior to studying Chapter 6 of *An Introduction to Galaxies and Cosmology*

When you are ready to start studying Chapter 6 of *An Introduction to Galaxies and Cosmology*, it is recommended that you view the ‘Summary’ section of the package.

- To start the package:
- Start the S282 Multimedia guide, and open the folder called ‘Cosmology’, then click on the icon for A history of the Universe.
- Click Start.

The first time you open the package you should view an introductory section which gives details of how to find information in this package.

- Click on the button labelled Introduction.

You should then view the ‘Summary’ sequence as this will set the scene for Chapter 6.

- Click on Summary.

(Altogether this should take about 10–15 minutes.)

2 During your study of Chapter 6 of *An Introduction to Galaxies and Cosmology*

You can return to the package during your study of Chapter 6 to get more information, or a different perspective on many topics that are described in the text.

After opening the package, use the red planet-shaped slider on the right hand side of the screen to select a period in the history of the Universe. (The time is shown in the top centre part of the screen.)

For instance, if you want to know more about inflation, move the slider to about 10^{-35} s. The word Inflation will appear on the main screen – click on this. The screen will then give you a choice of three topics about inflation

- What is inflation?
- What’s wrong with inflation?
- How does inflation gives rise to fluctuations?

So, by choosing the first of these topics, you will get the opportunity to view video clips in which five cosmologists (Alan Guth, Neil Turok, Jerry Ostriker, Andrew Liddle and Ed Copeland) give their explanation of what inflation is.

You will find that many of the topics covered in Chapter 6 of *An Introduction to Galaxies and Cosmology* are described in this package. Note also that the Search facility on the main screen of the package is another way to access descriptions of processes.

3 During your study of Chapter 8 of *An Introduction to Galaxies and Cosmology*

You will find that many of the more controversial areas of cosmology – which form some of the outstanding questions that are posed in Chapter 8 – are also discussed in this package. To return to the example discussed above, you might want to find out about some of the objections that cosmologists have to inflation. This is addressed in the section entitled 'What's wrong with inflation?', in which Martin Rees, Neil Turok and Ed Copeland raise different issues that they see as problematic for this key idea in cosmology.

There are many interesting items in the package. For instance you can hear Neil Turok describing what are, for him, the key challenges in cosmology, or Jerry Ostriker's views on the status of the anthropic principle.

4 As a resource of further information about galaxy formation and evolution

Since the package covers the entire history of the Universe it also contains information about the formation of galaxies, and so can be used to revise some ideas that were introduced in Chapter 2 of *An Introduction to Galaxies and Cosmology*. In particular, there are some interesting video clips (under the heading 'Galaxies evolve') in which Roger Davies describes recent developments in the study of the evolution of galaxies.